

# Moon Shadow Formula Calculation

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The Soudan paper on the moon shadow<sup>1</sup> used a formula equivalent to the following for the moon shadow:

$$\frac{dN_\mu}{d\Omega} = \lambda \left( 1 - \frac{R_M^2}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \right), \quad (1)$$

where  $\lambda$  is the average differential muon flux,  $R_M$  is the angular radius of the moon,  $r$  is the observed angular muon separation from the center of the moon, and  $\sigma$  is the rms smearing due to geomagnetic effects, multiple Coulomb scattering in the rock, and detector resolution. As explained in Ref. 1, Eq. 1 “treats the Moon as a point object at  $r = 0$  which removes  $\lambda\pi R_M^2$  muons from the sample.” The purpose of this note is to calculate the corrections to Eq. 1 to take into account the finite size of the moon.

Without the point-moon approximation, Eq. 1 clearly becomes

$$\frac{dN_\mu}{d\Omega} = \lambda \left( 1 - \int_0^{R_M} r' dr' \int_0^{2\pi} d\theta G(|\vec{r} - \vec{r}'|, \sigma) \right), \quad (2)$$

where  $G$  is a two-dimensional Gaussian convolution. Writing  $G$  out explicitly,

$$\frac{dN_\mu}{d\Omega} = \lambda \left( 1 - \frac{1}{2\pi\sigma^2} \int_0^{R_M} r' dr' \int_0^{2\pi} d\theta e^{-\frac{r^2 + r'^2 - 2rr' \cos \theta}{2\sigma^2}} \right). \quad (3)$$

Performing the  $\theta$  integration, we obtain

$$\frac{dN_\mu}{d\Omega} = \lambda \left( 1 - \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \int_0^{R_M} r' dr' I_0 \left( \frac{rr'}{\sigma^2} \right) e^{-\frac{r'^2}{2\sigma^2}} \right), \quad (4)$$

where  $I_0$  is a modified Bessel function of the first kind. The remaining integration in Eq. 4 cannot be done in closed form. However,  $I_0$  has a simple and rapidly decreasing Taylor expansion,

$$I_0(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{4^k (k!)^2} = 1 + \frac{x^2}{4} + \frac{x^4}{64} + \frac{x^6}{2304} \dots \quad (5)$$

Substituting the expansion and evaluating the integral, we obtain

$$\frac{dN_\mu}{d\Omega} = \lambda \left( 1 - \frac{R_M^2}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \left( 1 + \frac{(r^2 - 2\sigma^2)R_M^2}{8\sigma^4} + \frac{(r^4 - 8r^2\sigma^2 + 8\sigma^4)R_M^4}{192\sigma^8} + \dots \right) \right). \quad (6)$$

Using  $R_M = 0.26^\circ$  and  $\sigma = 0.41^\circ$ , in our region of interest the first and second correction terms provide  $r$ -dependent corrections of up to -10% and +0.7%, respectively.

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<sup>1</sup> E. H. Cobb et al., *Phys. Rev. D* **61**, 092002 (2000).