DIS Cross Section and Analysis Update

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- NEUGEN DIS Cross section Tuning/Checks
- Fake data studies (WIP)
  - ▷ Flux extraction (Low-ν flux method)
  - ▷ Total cross section
  - ▷ Differential cross section
- Cross section extracted from Data (WIP)
- Parameteric fit DIS model.
- Summary and Plans

DIS Model Tuning in NEUGEN

Recent work

- Correction to xF_3 (longitudinal structure function R_L).
  - ▷ Issue with PDFLIB-based PDF sets.
- Check of $r = \frac{\sigma^{\overline{\nu}}}{\sigma^{\nu}}$ (NEW models have decrease in r).
- Check of $R^{\nu} = \frac{\sigma(NC)}{\sigma(CC)}$ (NEW models have increase in R^{ν}).

R_L in Structure Functions

QPM Structure Functions in Neutrino Scattering

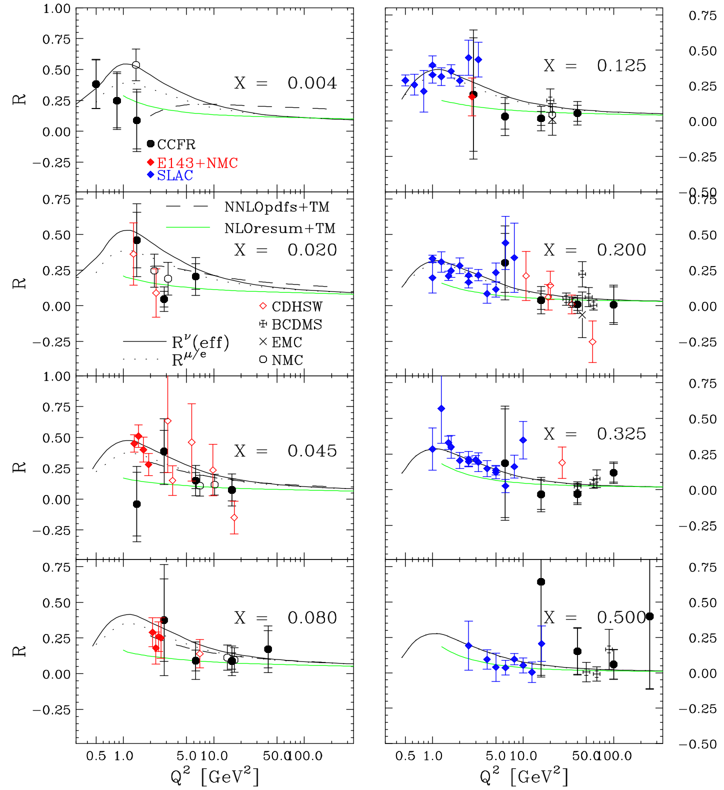
$$2xF_1^{\nu(\overline{\nu})} = \Sigma x(q+\overline{q}) = \sigma_T$$

$$F_2^{\nu(\overline{\nu})} = \Sigma x(q+\overline{q}+2k) = \sigma_T + \sigma_L$$

$$\approx 2xF_1(1+R)$$

$$xF_3^{\nu(\overline{\nu})} = \Sigma x(q-\overline{q}) \pm 2x(s-c)$$

- 2004 Default Neugen assumed $2xF_1 = F_2$, $R=0$.
- BYRSMOD, DUALITY include R_{World}



- Important for prediction of high y cross-section (Low E_μ).
- Important for low $Q^2 (< 5GeV^2)$ where R_L is non-negligible.

How to Include R_L in NEUGEN

In parton model, construct $2xF_1 = \Sigma(q + \overline{q})$, then include σ_L in F_2:

$$F_2^{\nu(\overline{\nu})} = \Sigma x(q + \overline{q} + 2k) = 2xF_1(x, Q^2)\left(\frac{1+R(x,Q^2)}{1+\frac{4M^2x^2}{Q^2}}\right)$$

(accounts for target mass)

- PDFLIB: CTEQ and MRST fit directly for $\Sigma(q(x, Q^2) + \overline{q}(x, Q^2))$ from $F_2(x, Q^2)$ data.
  $\Rightarrow R_L$ is incorporated in the quark distributions.
- Using PDFLIB: construct $F_2(x, Q^2) = \Sigma(q(x, Q^2) + \overline{q}(x, Q^2))$
- Remove R_L contribution from $2xF_1$ using: $2xF_1(x, Q^2) = F_2^{\nu(\overline{\nu})}\left(\frac{1+\frac{4M^2x^2}{Q^2}}{1+R(x,Q^2)}\right)$

  (This was done for the new release BYHTMOD-2, DUALITY)
- Also need to correct xF_3 to remove R_L now incorporated into the quark distributions.

  $$xF_3 \rightarrow xF_3 * \left(\frac{1+\frac{4M^2x^2}{Q^2}}{1+R(x,Q^2)}\right)$$

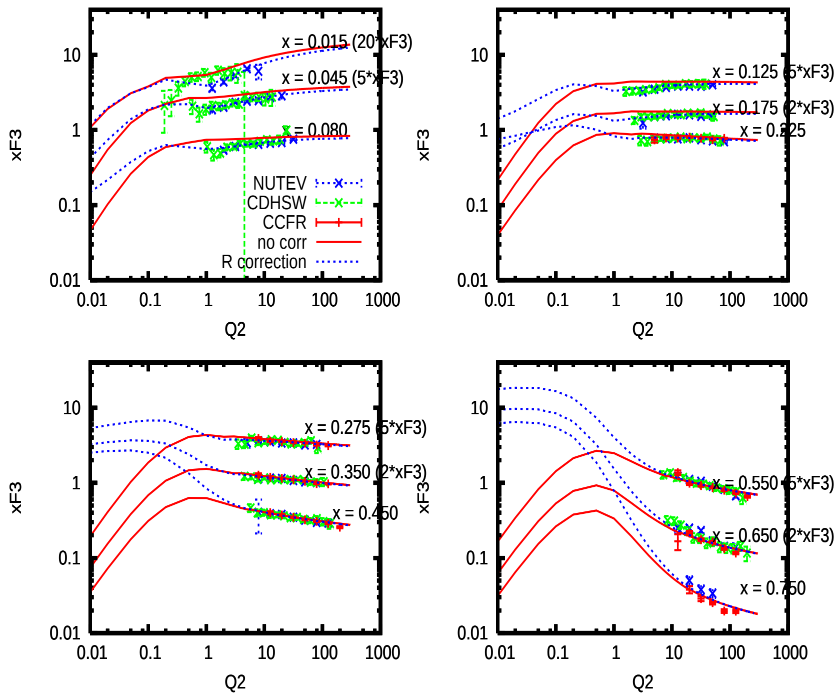
  (Will be included in future release BYHTMOD-3)

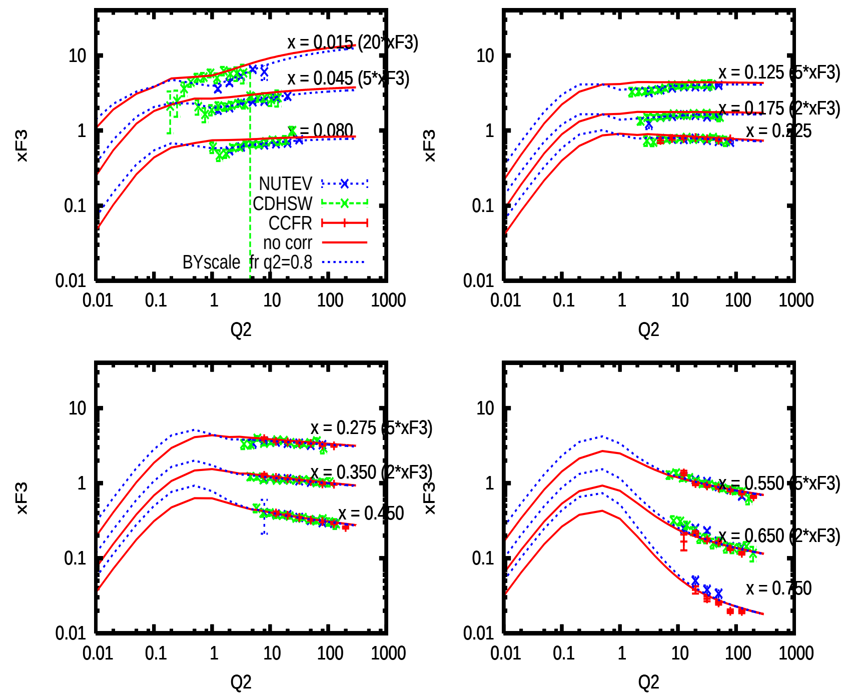
Modified $xF_3(x, Q^2)$

$$xF_3 \to xF_3 * \left(\frac{1 + \frac{4M^2 \xi_{BY}^2}{Q^2}}{1 + R(x, Q^2)} \right)$$

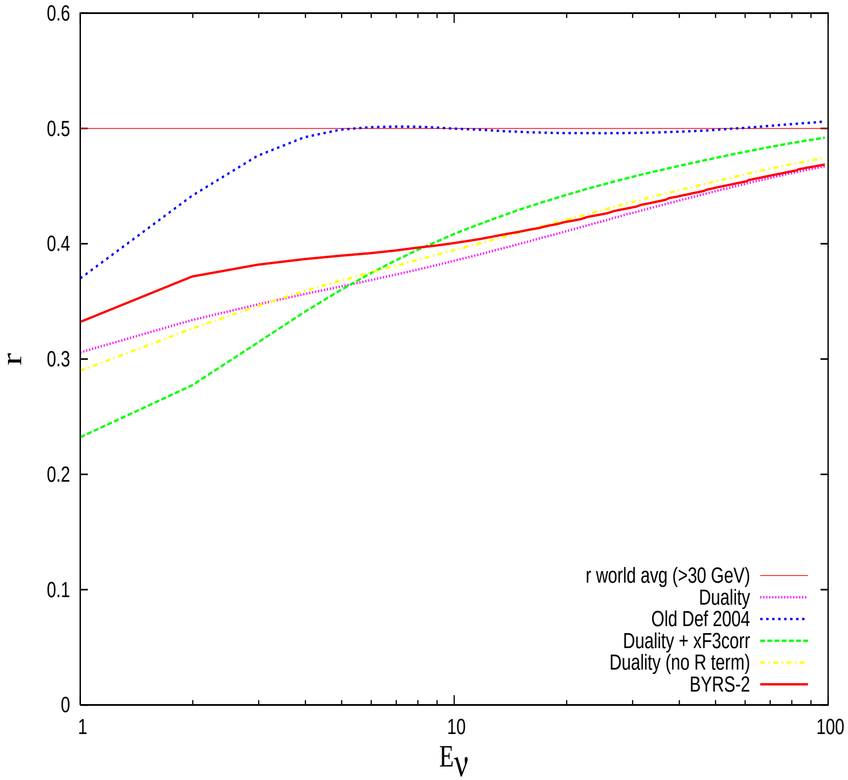
New xF_3:

Freeze Q^2 below 0.8 Gev^2 (same as BY model).



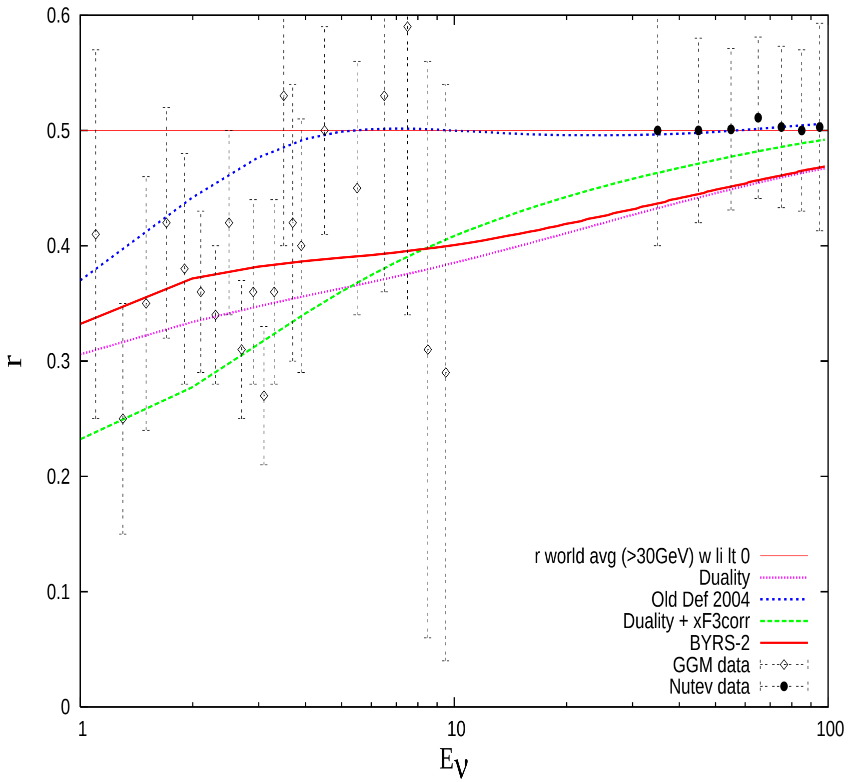


Check new $r = \frac{\sigma^{\overline{\nu}}}{\sigma^{\nu}}$



- New MC has lower $\overline{\nu}$ cross section.
- New models underestimate r (by about 10% at high energy r is uncertain at low energy- see next plot).
- r is intrinsic to the PDF set (cannot tune in BYMOD and DUALITY models).
  - ▷ Tuning $\frac{d}{u}$ may help.
  - ▷ $\frac{d}{u}$ is constrained by data

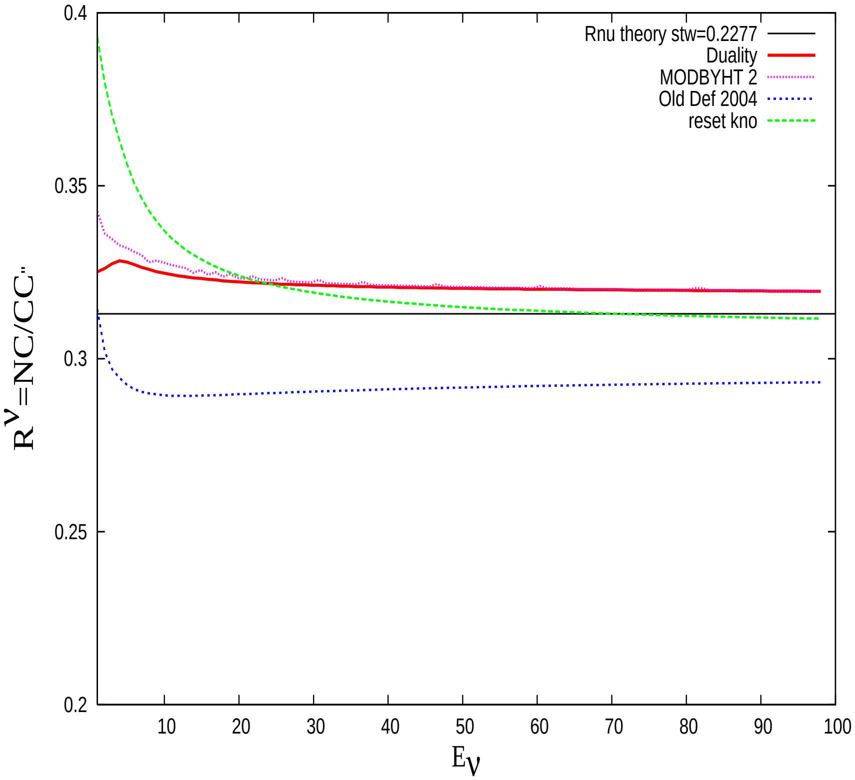
Comparison with Data



- Data is uncertain at low energy.
- New models systematically low at moderate and high energies (not clear at low energy).
- Including Rterm in xF3 (green curve) worsens agreement at low energy, improves agreement at high energy.

NC Cross Section in New Model

- NC cross section has increased by 10% (old default tuned KNO parameters separately for CC and NC, was about 10% low).
  - ▷ *NC and CC cross sections should not be independently tuned, they are related through the quark couplings.*
- Effect on $R^\nu = \frac{NC}{CC}$ of resetting KNO parameters in old default model (kno$_{cc}$=kno$_{nc}$ channel by channel).
- New models are close to theoretical value of R^ν (computed for $\sin^2\theta_w = 0.2277$).
  - ▷ *Particularly sensitive to charm mass value and strange sea. (may need additional tweaks to m_c and $s(x)$).*



Low ν Relative Flux Extraction Method

- Method valid for high E_ν, $(E > 5GeV)$
- Method does not separate exclusive QE channel to extract flux.
- Use inclusive low $\nu (= E_{\mathrm{SHRW}})$ cross section to get flux shape.

Start with general expression for differential cross section in terms of structure functions:

$$\frac{d^2\sigma^{\nu,\overline{\nu}}}{dx d\nu} = \frac{G^2 M}{\pi}\left[\left(1 - \frac{\nu}{E} - \frac{Mx\nu}{2E^2} + \frac{\nu^2}{2E^2}\frac{1+2Mx/\nu}{1+R}\right)F_2(x) \pm \frac{\nu}{E}\left(1 - \frac{\nu}{2E}\right)xF_3(x)\right]$$

Integrate over x for fixed ν:

$$\frac{d\sigma}{d\nu} = A\left(1 + \frac{B}{A}\frac{\nu}{E} - \frac{C}{A}\frac{\nu^2}{2E^2}\right)$$

$$\mathrm{A} = \frac{\mathrm{G^2M}}{\pi}\int \mathrm{F_2(x)dx}$$

$$\mathrm{B} = -\frac{\mathrm{G^2M}}{\pi}\int \left(\mathrm{F_2(x)} \mp \mathrm{xF_3(x)}\right)\mathrm{dx}$$

$$\mathrm{C} = \mathrm{B} - \frac{\mathrm{G^2M}}{\pi}\int \mathrm{F_2(x)}\left(\frac{1+\frac{2\mathrm{Mx}}{\nu}}{1+\mathrm{R(x)}} - \frac{\mathrm{Mx}}{\nu} - 1\right)\mathrm{dx}$$

- At low y, (i.e. low ν and high E_ν) $\Rightarrow (\frac{\nu}{E})$ and $(\frac{\nu}{E})^2$ terms are small.

$$\frac{d\sigma}{d\nu}^{\nu}_{\lim\, y\to 0} = \frac{d\sigma}{d\nu}^{\overline{\nu}}_{\lim\, y\to 0} = A$$

approaches a constant, independent of E_ν.

Low ν Flux Method (cont'd)

Integrate up to ν_o and apply corrections $\Phi(\mathrm{E}) \propto \int_0^{\nu_0} \left(\frac{\frac{dN(\mathrm{E})}{d\nu}}{1 + \frac{B}{A}\frac{\nu}{E} - \frac{C}{A}\frac{\nu^2}{2E^2}} \right) d\nu$

For small ν and high-E, corrections are small $\rightarrow$Relative flux given by: $\Phi(E_\nu) \propto N(E_\nu)_{(\nu<\nu_o)}$

- Correction of order $(\frac{\nu}{E})$ decreases with energy.
- Smaller for ν than for $\overline{\nu}$
  - - for neutrinos: $-1 < \frac{B}{A} < 0$
  - + for anti-neutrinos: $-2 < \frac{B}{A} < -1$

$$\frac{B}{A} = -\frac{\int (F_2(x) \mp xF_3(x))\, dx}{\int F_2(x)dx}$$

- Theoretical value for $\frac{B}{A}$ can be computed, (problem: likely to have large uncertainty at low ν)
  - $(\frac{B}{A})^\nu \approx -0.3$ (lower limit)
  - $(\frac{B}{A})^{\overline{\nu}} \approx -1.7$

- For MINOS use sample with $\nu < 1$ GeV, extract flux for $E_\nu > 5$ GeV
- for neutrinos, the correction is small above 5 GeV.
- Work on understanding theoretical uncertainty.

E_ν (GeV)	neutrino $\frac{B}{A}\frac{\nu}{E}$	anti-neutrino $\frac{B}{A}\frac{\nu}{E}$
5	3%	17%
10	1.5%	8.5%
20	0.75%	4.25%

- Next: test of the method using MC generated "Fake" data.

Fake Data Study of Flux Extraction

(D. Bhattacharya & M.S. Kim)

- Using R1.18 Monte Carlo divided into 2 samples; one sample used to obtain the acceptance correction, the other treated as “fake”data.

Selection of reconstructed sample

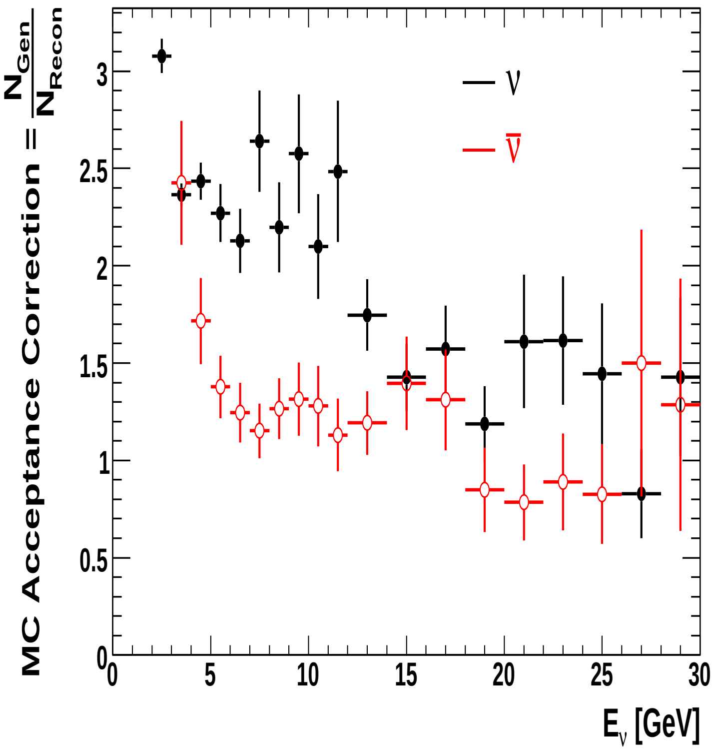
- ▷ Fiducial volume cuts (Vertex well contained in target region).
- ▷ Track and fit quality, $p_{\mathrm{ran}} > 1\mathrm{GeV}$
- ▷ Downstream stopping & exiting evts.
- ▷ $E_\mu > 2$GeV.
- ▷ $E_{\mathrm{SHWR}} < 1\mathrm{GeV}$

Selection of truth sample

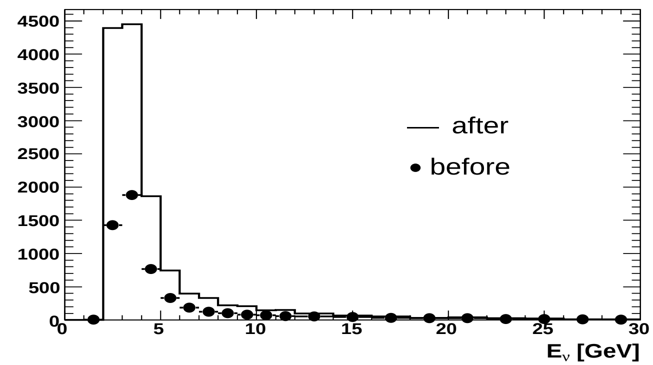
- ▷ Fiducial volume cuts (true vertex in target region, same geometry as for RECO sample).
- ▷ $E_{\mathrm{HAD}}(true) = yE_\nu < 1\mathrm{GeV}$

- Bin data in energy bins 5-6, ...12-14, ...28-30 GeV
- Correct the data using acceptance correction, $\frac{\mathrm{gen}}{\mathrm{reco}}$
  - ▷ $N_{\mathrm{CORR}}(E, \nu < 1) = N_{\mathrm{RECO}}(E, \nu < 1) * \frac{N^{MC}_{\mathrm{gen}}(E,\nu<1)}{N^{MC}_{\mathrm{reco}}(E,\nu<1)}$
- Compare with known input flux for R1.18 (V17).

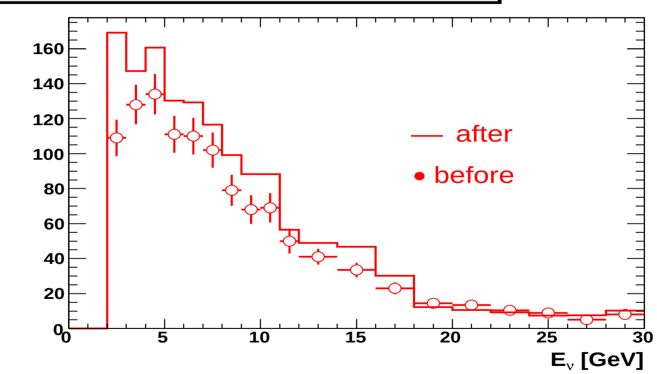
Fake Data Study of Flux Extraction (cont'd)



Before and After MC Acceptance Correction for Φ sample of ν

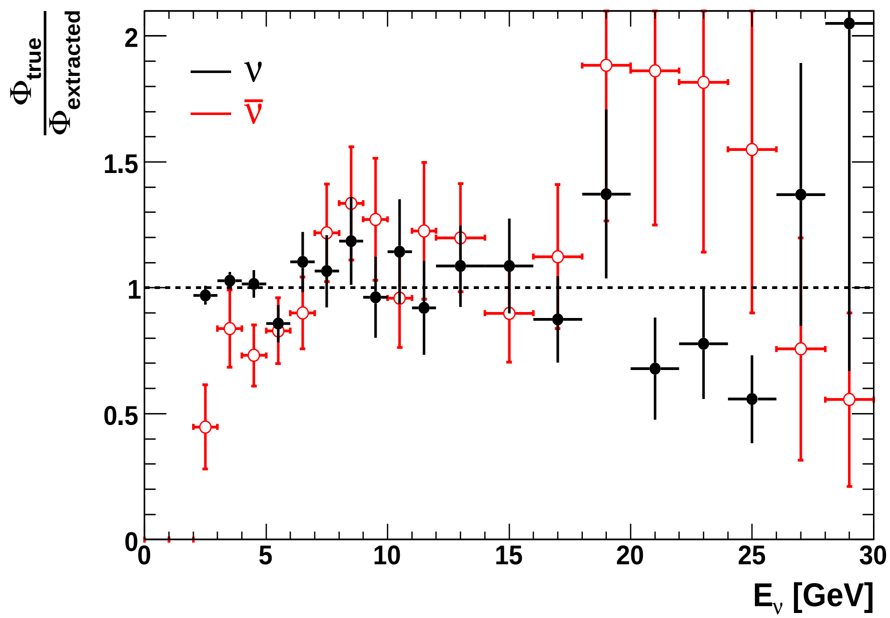


Before and After MC Acceptance Correction for Φ sample of $\bar{\nu}$



Result: Fake Data Flux Extraction

- Extracted flux: normalized to input flux above 5 GeV (shape only).
  - ▷ For data, normalization of flux comes from high-energy world average $\frac{\sigma}{E}$.
- MC sample corresponds to 1.8E18 POT.
- No $\frac{B}{A}$ correction applied. (important for $\overline{\nu}$)



Fake Data Study: Total Cross Section Sample

(D. Bhattacharya & M.S. Kim)

- Needed to normalize the flux (see also work by Mike K.)
- Minos can measure $r = \frac{\sigma^{\overline{\nu}}}{\sigma^{\nu}}$ in moderate energy range where data is sparse.

- Using R1.18 Monte Carlo divided into 2 samples; one sample used to obtain the acceptance correction, the other treated as "fake"data.

<u>Selection of reconstructed sample</u>

- ▷ Fiducial volume cuts (Vertex well contained in target region).
- ▷ Track and fit quality, $p_{\rm ran} > 1{\rm GeV}$
- ▷ Downstr. exiting and stopping only.
- ▷ $E_{\mu} > 2$GeV.

<u>Selection of truth sample</u>

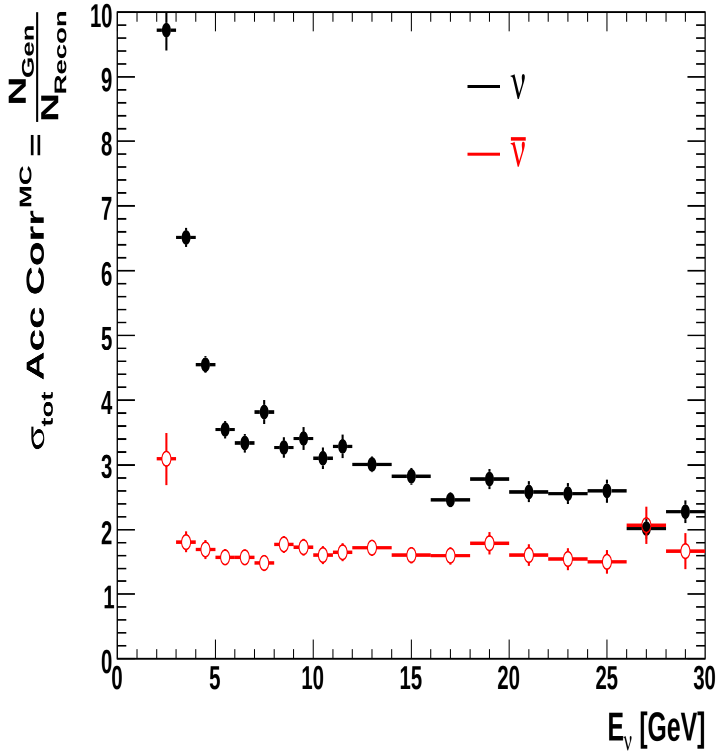
- ▷ Fiducial volume cuts (true vertex in target region, same geometry as for RECO sample).

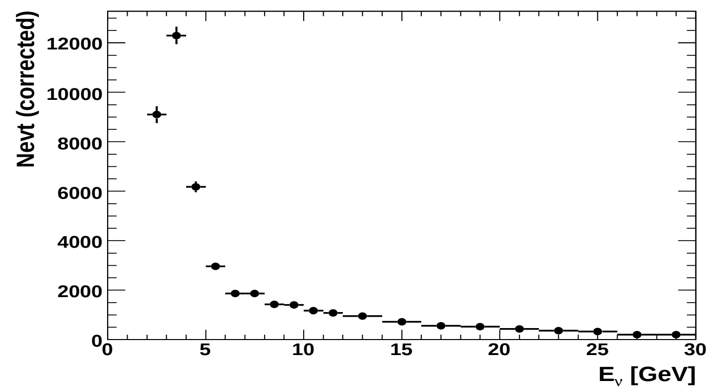
- Correct the binned data using acceptance correction, $\frac{\rm gen}{\rm reco}$

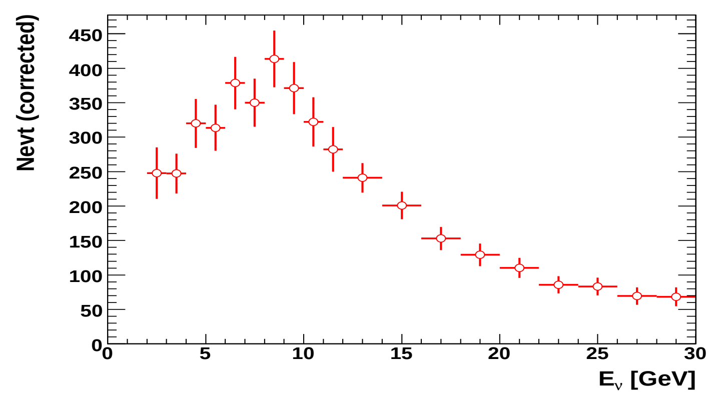
  ▷ $N^{xsec}_{\rm CORR}(E) = N^{xsec}_{\rm RECO}(E) * \frac{N^{MC xsec}_{\rm gen}(E)}{N^{MC xsec}_{\rm reco}(E)}$

- Divide $N^{xsec}_{\rm CORR}(E)$ by extracted flux in energy bins. ▷ $\sigma_{\rm TOT} = \frac{N^{xsec}_{\rm CORR}(E)}{\Phi(E)}$

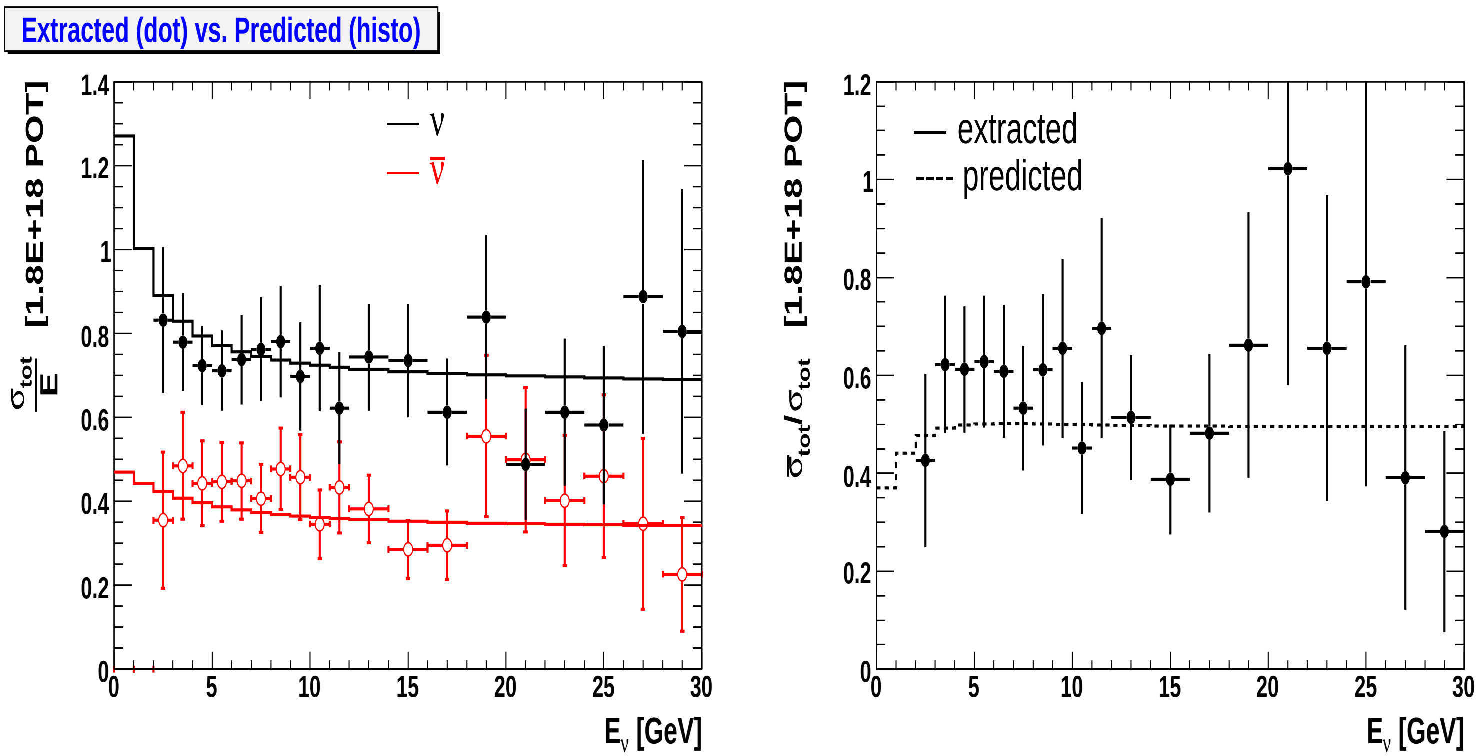
Fake Data Study: Total Cross Section Sample (cont’d)







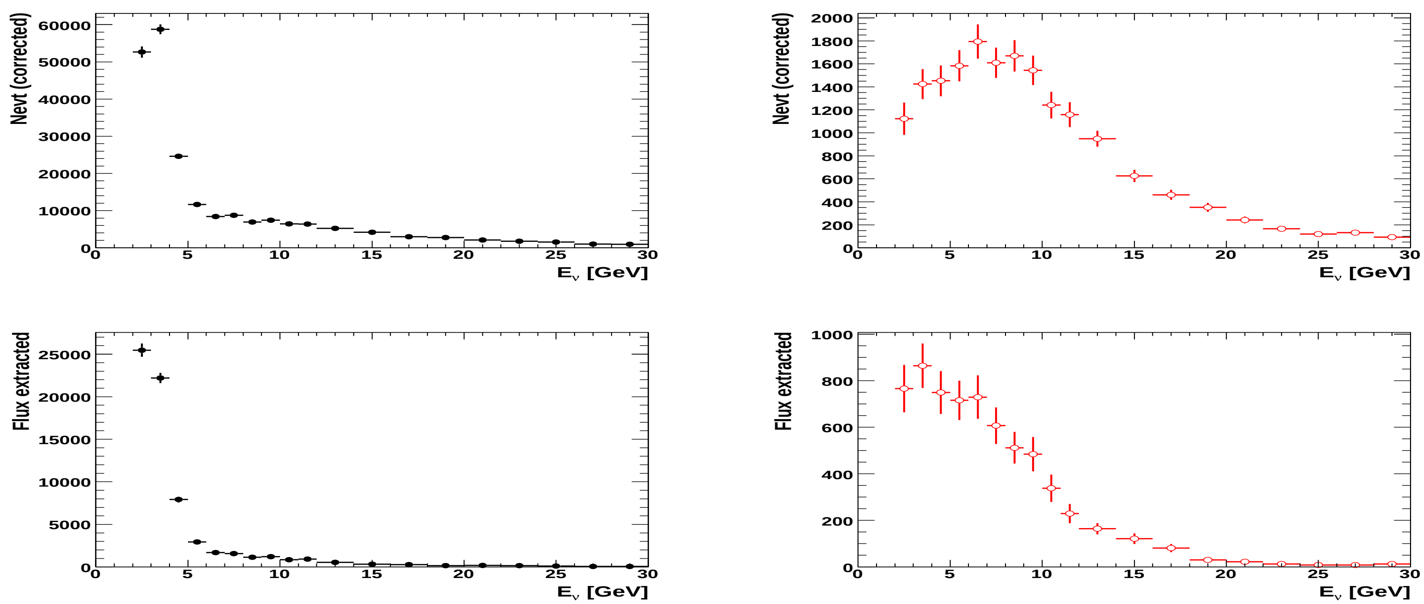
Fake Data Study: $\frac{\sigma}{E}$



- MC sample 1.8E18 PoT (same for 'fake data' and for acceptance correction).
  ▷ Stat. errors on the flux sample dominate the error bars.
- Energy dependence of cross section agrees with NEUGEN pred.(input). $\chi^2_\nu = 0.3$, $\chi^2_{\overline{\nu}} = 0.4$, $\chi^2_r = 1.3$ ▷ Caveat: $\overline{\nu}$ should not be taken seriously below $10\ GeV$.

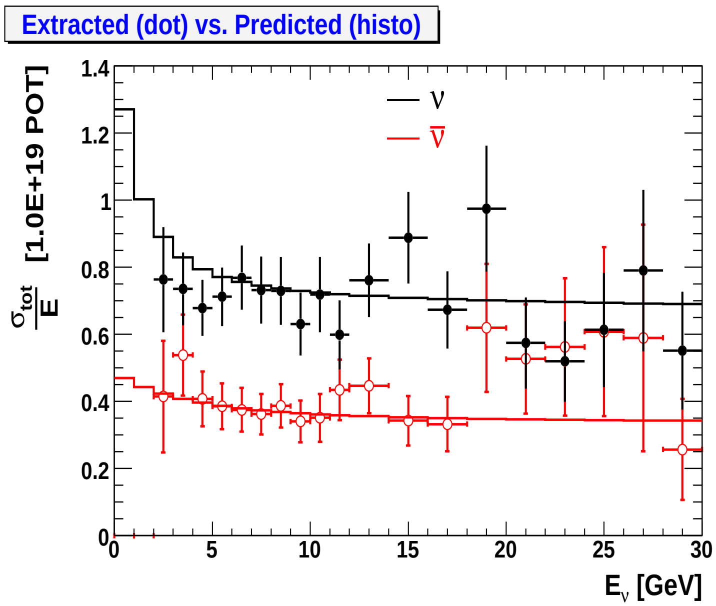
Data: Total Cross Section Sample (WIP)

- Use June data (1E19 POT).
  - ▷ Cross Section and Flux Sample: Good Muon in fiducial volume with $E_\mu > 2\,\mathrm{GeV}$.
  - ▷ Flux sample only: additional requirement $E_{\mathrm{shwr}} < 1\,\mathrm{GeV}$.
- Correct the binned data using acc. corr., $\frac{\mathrm{gen}}{\mathrm{reco}}$ [from R1.18 MC]



- Divide $N^{xsec}_{\mathrm{CORR}}(E)$ by extracted flux in energy bins: $\sigma_{\mathrm{TOT}} = \frac{N^{xsec}_{\mathrm{CORR}}(E)}{\Phi(E)}$

Result: Energy Dependence of Cross Section

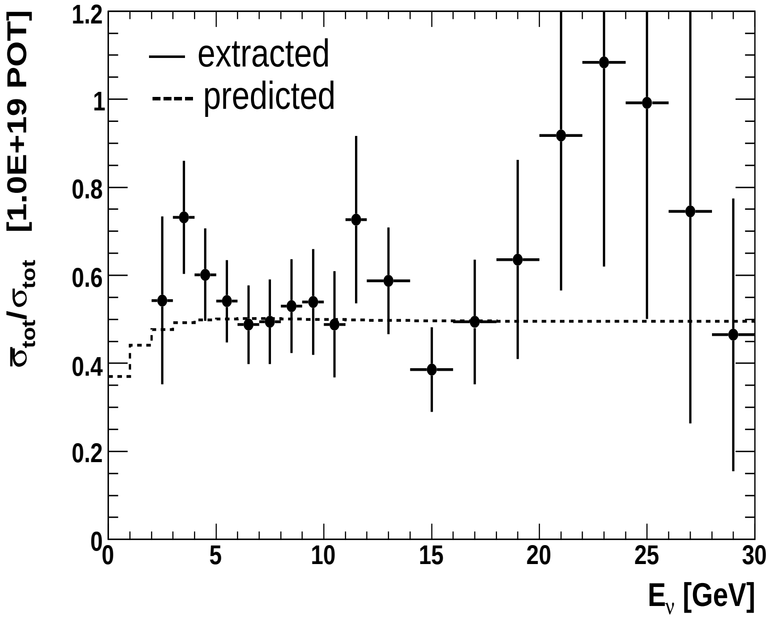


- Normalization factor applied to flux so that average neutrino cross section level is world average above 10 GeV.
- Measures the shape of the cross section with energy (agrees with Neugen) $\chi^2_\nu = 0.4$, $\chi^2_{\overline{\nu}} = 0.6$.
- Same normalization applied to $\overline{\nu}$ flux. $\Rightarrow$ measures $r = \frac{\sigma^{\overline{\nu}}}{\sigma^{\nu}}$.
- Caveat: $\overline{\nu}$ flux does not have $\frac{B}{A}$ correction - important at low energy (this will decrease $\frac{\sigma}{E}$ at low energy).

▷ Error bars dominated by acceptance correction based on 1.8E18 PoT.
  - ▷ First, need more MC statistics. (10× data statistics).
  - ▷ Next: look at higher statistics data sample.

▷ Acceptance correction computed using R1.18 (old cross section model/Flux)
  - ▷ Update to new MC for acceptance correction.

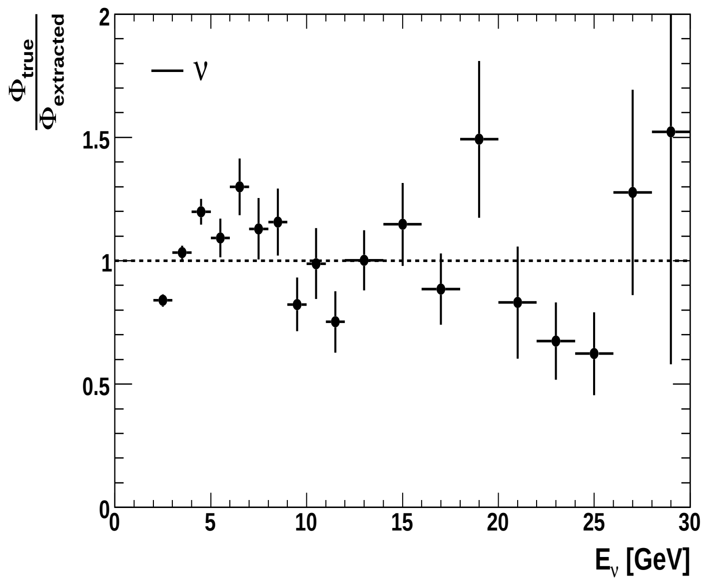
Result: Ratio of Anti-neutrino to Neutrino Cross Section

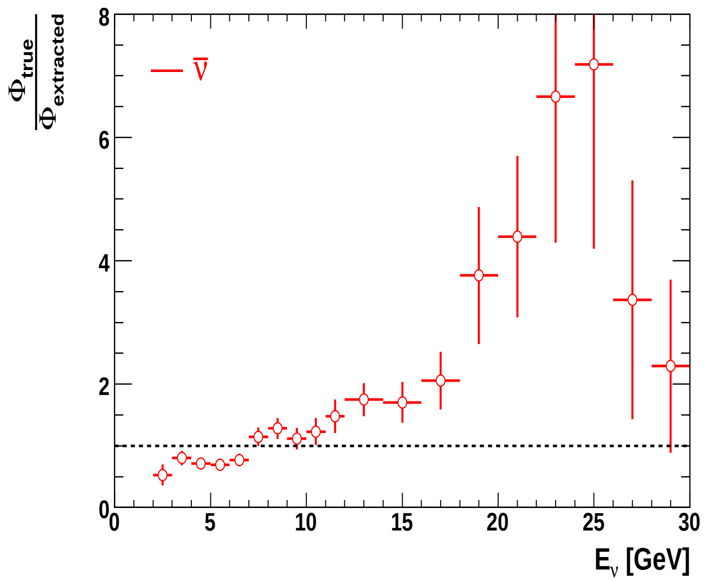


- Preliminary r measured, (flux normalization cancels). $\chi^2 = 2.0$.
  - Independent of GNUMI flux, uses low-ν flux.
  - Apply $\frac{B}{A}$ correction will increase $\overline{\nu}$ Flux at low energy $\Rightarrow$ decrease $\sigma^{\overline{\nu}}$.

Measured Flux Shape Comparison

- Ratio of beam flux (V17) to low-ν extracted flux. (Normalized by area for $E_\nu > 5$GeV).





Method valid for high-energy tail region:

- ν flux in tail agrees with GNUMI prediction shape.
- $\overline{\nu}$ GNUMI flux is too high in tail (by 2-6$\times$).
  - ▷ Need $\frac{B}{A}$ correction for $\overline{\nu}$, (below 10 GeV).

Fake Data Study: Differential Cross Section

Selection of reconstructed sample

- Fiducial volume cuts (Vertex well contained in target region).
- Track and fit quality, $p_{\rm ran} > 1\rm GeV$
- Downstr. exiting and stopping only.
- $E_\mu > 1$GeV.
- $E_{\rm SHWR} > 1\rm GeV$, $Q^2 > 1\rm GeV^2$

Selection of truth sample

- Fiducial volume cuts (true vertex in target region, same geometry as for RECO sample).

- Bin data in x, y, and E bins.
- Apply acceptance correction, $\frac{\rm gen}{\rm reco}$
  - Remove bins with $\frac{\rm reco}{\rm gen} < 10\%$.

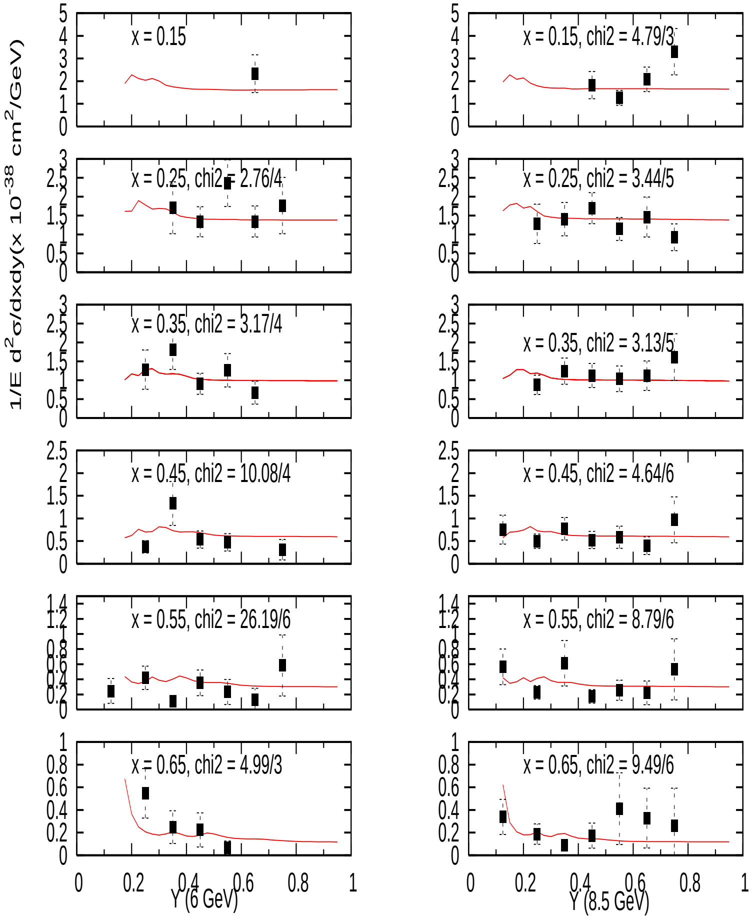
$$N_{\rm CORR}(x_i, y_j, E) = N_{\rm RECO}(x_i, y_j, E) * \frac{N_{\rm gen}(x_i, y_j, E)}{N_{\rm reco}(x_i, y_j, E)}$$

- Divide by the extracted flux

$$\frac{d^2\sigma}{dx\,dy} = \frac{N_{\rm CORR}(x_i, y_j, E)}{\Phi E}$$

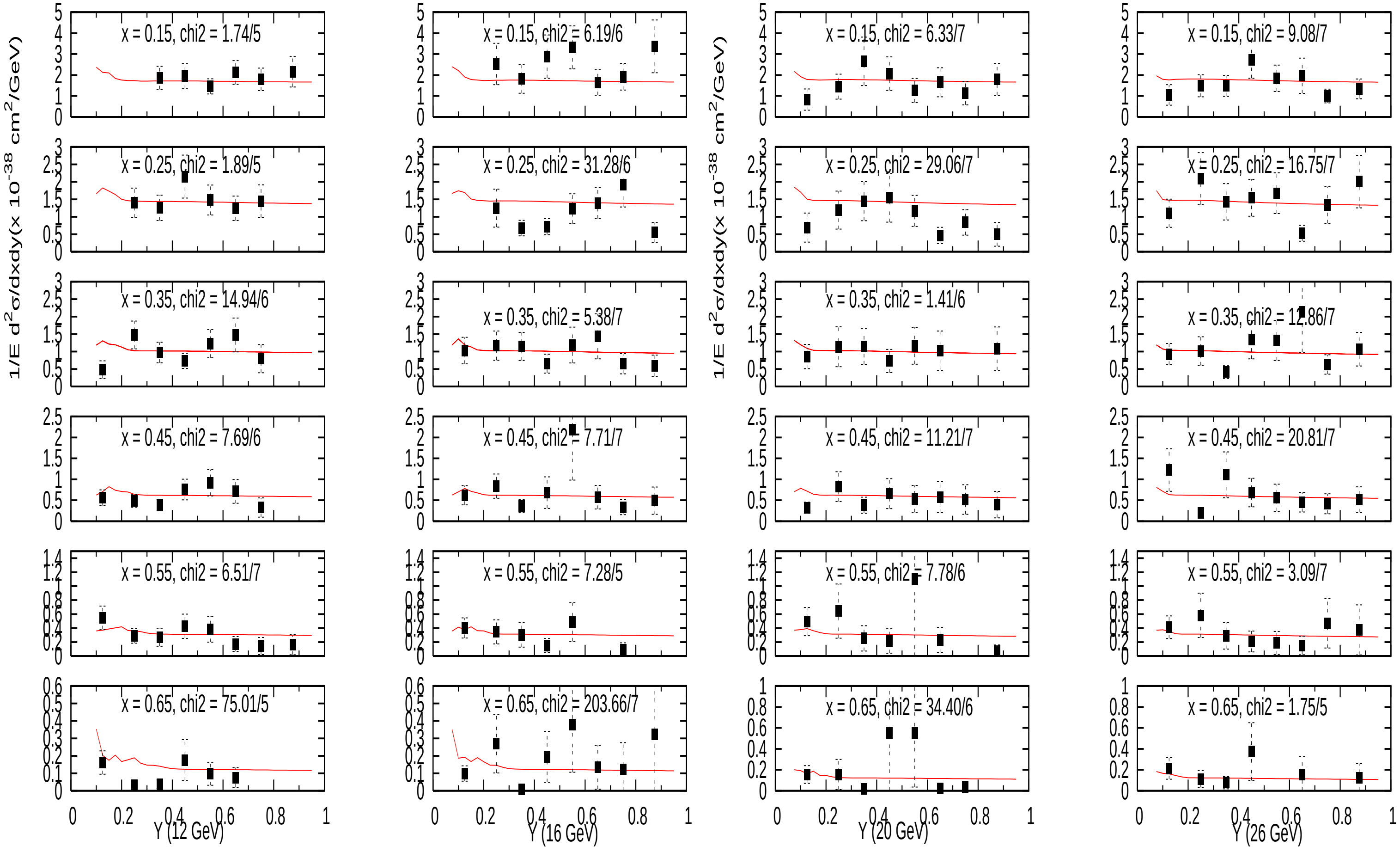
no.	E (GeV)	y bin	x bin
1	5-7	0.05-0.20	0.01-0.1
2	7-10	0.20-0.30	0.1-0.2
3	10-14	0.30-0.40	0.2-0.3
4	14-18	0.40-0.50	0.3-0.4
5	18-22	0.50-0.60	0.4-0.5
6	22-30	0.60-0.70	0.5-0.6
7		0.70-0.80	0.6-0.7
8		0.80-0.95	0.7-0.8
9			0.8-0.9

Fake Data Study: Differential Cross Section (cont'd)



- Need higher statistics acceptance corrections.
- Goal: Measure differential cross section.
  - ▷ Use for parametric model fit (next topic).
    - ⋆ Currently, there is only one tunable parameter in the DIS model 'DISFACT'.
  - ▷ Iterate until data/MC agreement is good.

Fake Data Study: Differential Cross Section (cont'd)



Fitting DIS Cross Section

- Cross section analysis- sensitivity to cross section model through acceptance corrections.
  - To be confident of acceptance corrections requires that MC models the data well.
- Ultimately, cannot trust PDFLIB based model in our kinematic region (+ unknown nuclear corr.).
- Custom model based-on fit to extracted differential cross section.
  - ⋆ Inclusive variables E, y, E_μ, E_{HAD} far less sensitive to pdf's.

Buras-Gaemers Parameterization - [A. J. Buras and K. L. F. Gaemers, Nucl. Phys. **B 132** (1978) 2109.]

Valence quark densities:
$xu_v(x, Q^2) = A_v(s)x^{E_1(s)}(1-x)^{E_2(s)}$
d_v quark density: $xd_v(x, Q^2) = c_d(s)x^{E_1(s)}(1-x)^{E_2(s)+1}$ falls more rapidly as $x \to 1$.
$c_d(s)$ is not independent, it is related to $A_v(s)$ through the quark counting rule:

$$\int u_v(x, Q^2)dx = 2\int d_v(x, Q^2)dx$$

Where $s = log\left(\frac{log(Q^2/\Lambda^2}{log(Q_o^2/\Lambda^2}\right)$, Q_o^2 is arbitrarily chosen in the experiment's accessible Q^2 range.

$E_1(s) = E_{10} + sE_{11}$
$E_2(s) = E_{20} + sE_{21}$

$A_v, \Lambda, E_{10}, E_{11}, E_{20}, E_{21}$ are the free fit parameters for the valence density.

Buras-Gaemers Parameterization (cont'd)

Light quark sea quark densities, $\overline{u}(x,Q^2)$=$\overline{d}(x,Q^2)$= $xs(x,Q^2) = A_s(Q^2)(1-x)^{E_s(Q^2)}$
The Q^2 dependence is given by the moments of the sea distribution which depend on the variable s

$$A_s(Q^2) = \langle x\mathbf{s}(Q^2)\rangle_2 \left(\frac{\langle xs(Q^2)\rangle_2}{\langle xs(Q^2)\rangle_3} - 1 \right)$$

$$E_s(Q^2) = \frac{\langle xs(Q^2)\rangle_2}{\langle xs(Q^2)\rangle_3} - 2$$

The moments are given by:

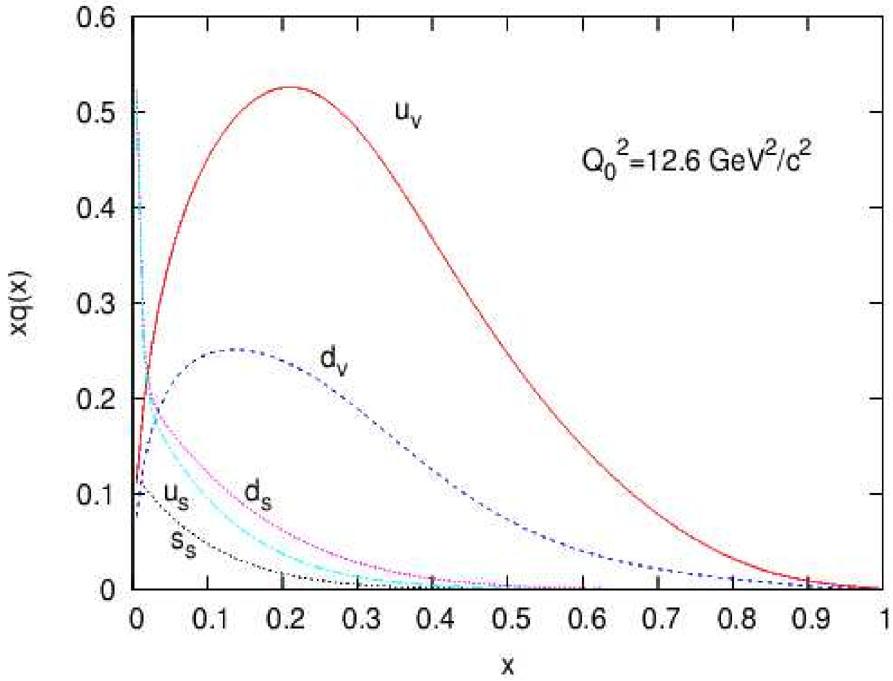
$$\langle xs(Q^2)\rangle_n = \int_0^1 dx\, x^{n-2} xs(x,Q^2)$$

- There are two independent parameters that define the sea, $A_s(Q_o^2)$, $E_s(Q_o^2)$
  - ▷ The strange sea is input from measurements and not part of the fit.
  - ▷ More parameters were needed to define the sea in NuTeV and may be added.

The gluon distribution has the same form: $xg(x,Q^2) = A_g(s)(1-x)^{E_g}$
The parameters are not independent, $\langle xg(Q^2)\rangle_2$ is obtained from the momentum sum rule as missing momentum at $Q^2 = Q_o^2$, and $\langle xg(Q^2)\rangle_3$ is the remaining fit parameter.

- *Modified version has 9 fit parameters in total.*
  - ▷ *Need to constrain the fit in regions outside our acceptance where data exists (incorporate external data).*
  - ▷ *Must incorporate all the pdf constraints.*
  - ▷ *May need to add parameters, esp. to constrain the sea.*

NuTeV Fitted pdf's



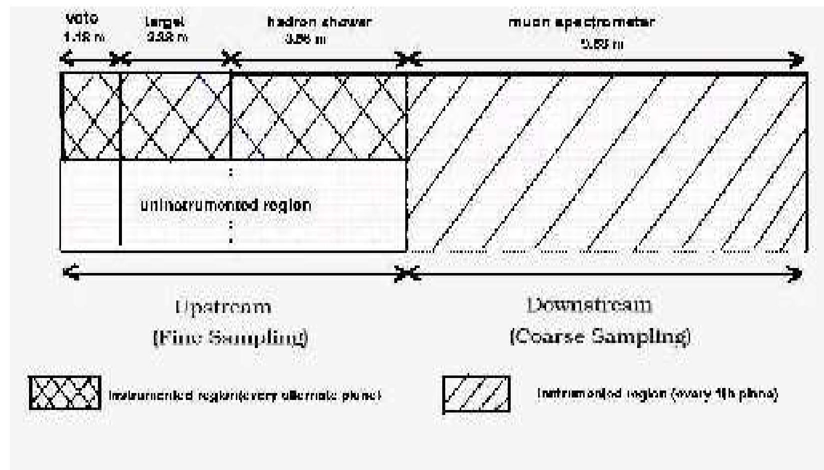
Summary and Plans

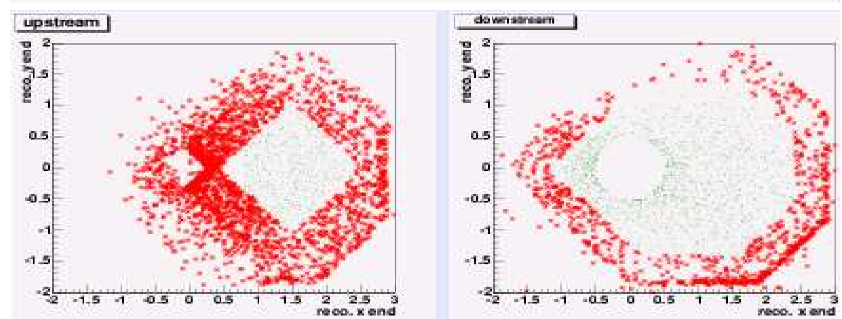
- Small tweaks to NEUGEN DIS cross section model are ongoing.
  - ▷ Look at $s(x)$, $\overline{s(x)}$, m_c for effect on total cross section and NC cross section.
  - ▷ Tuning of r, using $\frac{d}{u}$, (consult with A. Bodek).
- Fake data studies close (EXTRACTED= INPUT) for flux extraction (ν), Total cross section, Differential cross section.
  - ▷ $\frac{B}{A}$ correction for $\overline{\nu}$ flux needed, understand theoretical uncertainty.
- ν and $\overline{\nu}$ cross section shape with energy measured ($E > 5$GeV).
  - ▷ New acceptance corrections: need to use new MC version for xsec/flux.
  - ▷ Need increased MC statistics to reduce the uncertainty in the acceptance correction.
  - ▷ Next, implement parameteric fit model and reweighting.

(D. Bhattacharya)

Selecting DIS Sample:

- Track and shower vertex matching: Within 5 planes in z, 15 cm in x and y.
- vertex well contained in 'target' region, $0.6m < vz < 3.56m$, distance from transverse edge $0.5m$.
- Good muon track with $E_\mu > 1$GeV,
- Min. ν cut: $E_{shw} > 1GeV$
- Minimum energy $E_\nu > 5GeV$
- Minimum $Q^2 > Q_o^2$





- Four samples: upstream stopping, upstream exiting, downstream stopping, downstream exiting.