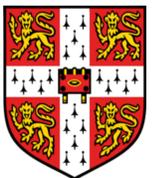


Mass Hierarchy Study with MINOS Far Detector Atmospheric Neutrinos



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Neutrino Mass Hierarchy

Neutrino Flavours



Neutrino Masses



It is experimentally firmly established that neutrinos have three flavours ν_e, ν_μ, ν_τ (and their anti-particles), and three masses (m_1, m_2, m_3). Flavour determines how they take part in the weak interaction, mass determines how they propagate through space.

From the neutrino oscillation data, in absolute value, one of the two independent neutrino mass squared differences Δm_{21}^2 is much smaller than the second one Δm_{32}^2

$$\Delta m_{21}^2 = m_2^2 - m_1^2 = 7.59 \times 10^{-5} \text{ eV}^2$$

$$|\Delta m_{32}^2| = |m_3^2 - m_2^2| = 2.32 \times 10^{-3} \text{ eV}^2$$

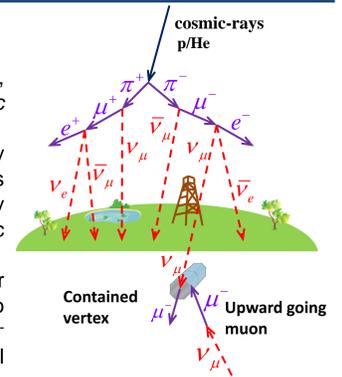
One refers to $\Delta m_{32}^2 > 0$ as normal mass hierarchy and $\Delta m_{32}^2 < 0$ as inverted mass hierarchy. Its determination remains one of the outstanding problems in neutrino physics, and is the focus of this poster.

Capture Atmospheric Neutrinos with MINOS Detector

Primary cosmic ray (mainly protons, with a small fraction of heavier nuclei) interacts with atmosphere nuclei to generate secondary cosmic rays (hadrons and their decay products). The produced pions and kaons decay into muons, muon neutrinos, electrons, electron neutrinos. These neutrinos are *atmospheric neutrinos*.

The deeply underground located 5.4 kton MINOS far detector can be used to study atmospheric neutrinos and antineutrinos oscillations. Its underground location and its scintillator veto shield constructed above the detector enables MINOS to study atmospheric neutrinos with greatly reduced background against the high flux of cosmic rays.

The MINOS far detector is the first large underground magnetized detector providing charge sign identification of muons produced by charged current neutrino interactions, enabling neutrinos and antineutrinos separation. By comparing μ^+ and μ^- event rates in different neutrino mass hierarchy hypothesis, MINOS has the potential ability to determine the mass hierarchy.

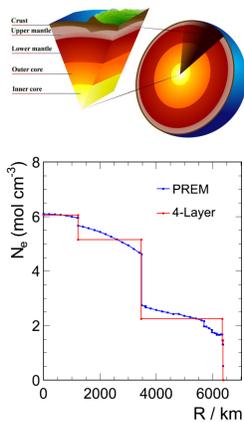


Neutrino Oscillations and Matter Effects

Neutrino oscillation probabilities are significantly modified by coherent forward scattering from electrons in matter when propagation through the Earth. This *matter effects* predominantly depend on the neutrino energy, the neutrino trajectory zenith angle, the electron number density of the earth medium, the atmospheric mass squared splitting Δm_{32}^2 , and the mixing angles θ_{13}, θ_{23} .

Matter effects alter neutrino and antineutrino oscillations differently, the sign of Δm_{32}^2 determines whether neutrino or antineutrino oscillation probabilities are enhanced or suppressed. This matter induced neutrino-antineutrino asymmetry causes neutrino induced muon charge asymmetry.

Median electron density in each region of preliminary reference Earth model (PREM) is implemented in MINOS Monte Carlo simulation. Spherically symmetric distributed piecewise constant radial matter density is modeled. The oscillation probability is calculated with the product of the transition amplitudes of each layer a neutrino travels together with the amplitude crossing the atmosphere.



Detector Resolution and Optimal Binning

Reconstructed Neutrino Energy Resolution:

Bins (GeV)	0-2	2-4	4-6	6-8	8-10	>10
$\delta E/E$	0.16	0.15	0.14	0.15	0.14	0.16

Reconstructed Muon Track Angular Resolution:

Bins (GeV)	0.5 - 1	1 - 2	2 - 3	3 - 5	5 - 8	> 8
$\theta_{\text{reco}} - \theta_{\text{true}}$ (degree)	23.7	18.0	13.0	9.9	7.6	5.9

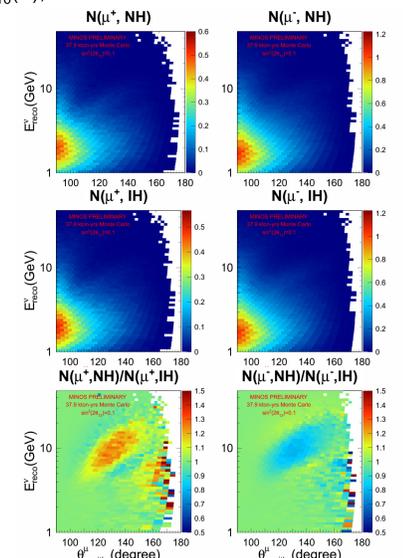
Optimized Binning Scheme :

- 56 bins in $\log_{10}(E)$ from 0 to 1.68 ($1 < E < 48$ GeV).
- Increase number of bins in θ_{zenith} linearly with $\log_{10}(E)$, from 24 to 79 bins.

The choice of optimal binning scheme in the two dimensional reconstructed energy and reconstructed zenith angle is to keep the bin size close to half of angular resolution over the full energy range.

Two categories of atmospheric neutrino events are observed in the MINOS far detector, based on the neutrino interaction position inside the detector (contained vertex neutrinos) or in the rock (upward going muons). 2553 live-days (37.9 kton-years) of data contained vertex neutrino are accumulated for the analysis. The statistical sensitivities of contained vertex neutrinos with simulated 193,240 kton-years Monte Carlo events are studied.

Figures on the right shows the Monte Carlo reconstructed zenith angle vs reconstructed energy with $\sin^2 2\theta_{13} = 0.1$ for normal hierarchy (NH) and inverted hierarchy (IH), and for μ^+ and μ^- samples separately. The event rate ratio between two mass hierarchy hypotheses demonstrates the neutrino-antineutrino asymmetry from the matter effects.



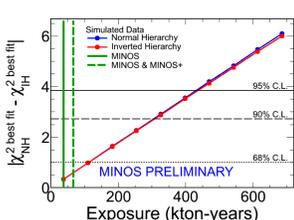
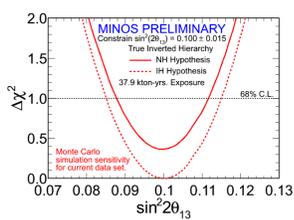
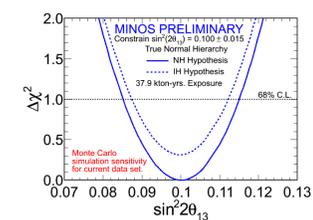
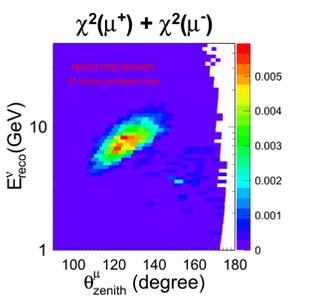
Mass Hierarchy Sensitivity Estimation

The mass hierarchy determination sensitivity is estimated with different analysis technique, before data is examined:

- χ^2 difference between two hypotheses, $\Delta\chi^2 = \chi^2_{\text{NH}} - \chi^2_{\text{IH}}$.
- The probability to determine the correct mass hierarchy.
- The "discrimination power" to determine the correct mass hierarchy.

$\Delta\chi^2$ Analysis

193,240 kton-years Monte Carlo contained vertex neutrinos events are generated for each set of oscillation parameters to ensure the statistical error is negligible, then they are normalized to current MINOS atmospheric neutrino 37.9 kton-years equivalent data exposure. Total $\chi^2 [= \chi^2(\mu^+) + \chi^2(\mu^-)]$ is calculated both for NH and IH. χ^2 is marginalized over $\sin^2 2\theta_{13}$ for both NH and IH with μ^+ and μ^- separately for a given set of input data $\sin^2 2\theta_{13} = 0.1$. The χ^2 difference $\Delta\chi^2 = \chi^2_{\text{NH}} - \chi^2_{\text{IH}}$ provides an estimation of mass hierarchy sensitivity. $\Delta\chi^2$ is roughly 0.3 for MINOS 37.9 kton-years contained vertex neutrinos events, and increases to around 0.6 for expected 66.5 kton-years of total exposure for the planned MINOS+ together with MINOS data.

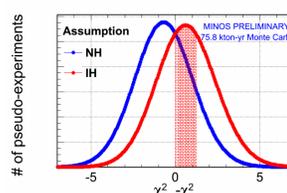
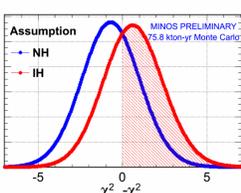
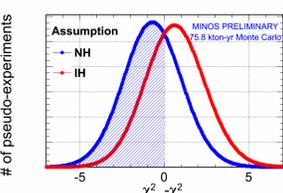


The probability to determine the correct mass hierarchy

Before looking at the data, the following strategy always gives better (> 50%) chance to choose the correct hierarchies: $\Delta\chi^2 < 0$, choose NH; $\Delta\chi^2 > 0$, choose IH. The current MINOS 37.9 kton-years contained vertex neutrino events give, on average, the probability of 60.5% to determine the correct mass hierarchy.

The "discrimination power" to determine the correct mass hierarchy

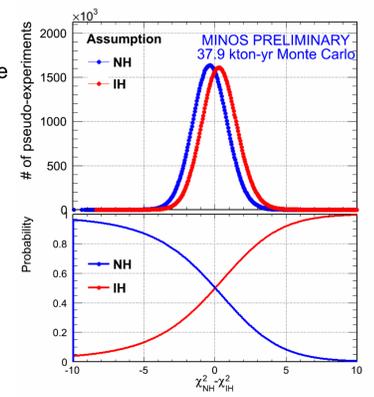
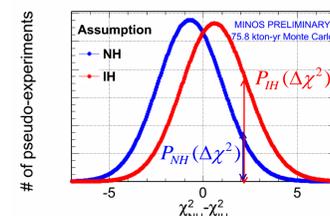
For a given mass hierarchy hypothesis, an interval for the histogram of $\chi^2_{\text{NH}} - \chi^2_{\text{IH}}$ is filled from the most probable $\chi^2_{\text{NH}} - \chi^2_{\text{IH}}$ to the less probable, until one side reaches the wrong determination region. This *discrimination power* is zero if most probable is zero, 100% if the entire histogram is on the correct side. The current MINOS 37.9 kton-years contained vertex neutrino events give 24.2% for NH, and 19.1% for IH, the *discrimination power* to determine the correct mass hierarchy



Measurement of the Mass Hierarchy

A statement may be made after the data sample is looked at, on "the probability to be normal mass hierarchy or inverted mass hierarchy" once $\chi^2_{\text{NH}} - \chi^2_{\text{IH}}$ is measured based on the Bayes' theorem.

$$P(\text{IH} | \Delta\chi^2) = \frac{P(\Delta\chi^2 | \text{IH})}{P(\Delta\chi^2 | \text{IH}) + P(\Delta\chi^2 | \text{NH})}$$



Current and Future Sensitivity

The probability to determine the correct mass hierarchy

Exposure (kton-years)	Normal Mass Hierarchy	Inverted Mass Hierarchy	Average
37.9	61.3%	59.7%	60.5%
75.8	65.6%	63.9%	64.7%
379	80.6%	79.1%	79.8%

The "discrimination power" to determine the correct mass hierarchy

Exposure (kton-years)	Normal Mass Hierarchy	Inverted Mass Hierarchy
37.9	24.2%	19.1%
75.8	31.4%	27.6%
379	61.3%	57.5%