Moon Shadow Formula Calculation

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The Soudan paper on the moon shadow[1] used a formula equivalent to the following for the moon shadow:

$$\frac{dN_\mu}{d\Omega} = \lambda\left(1 - \frac{R_M^2}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}}\right), \tag{1}$$

where λ is the average differential muon flux, R_M is the angular radius of the moon, r is the observed angular muon separation from the center of the moon, and σ is the rms smearing due to geomagnetic effects, multiple Coulomb scattering in the rock, and detector resolution. As explained in Ref. 1, Eq. 1 "treats the Moon as a point object at $r = 0$ which removes $\lambda\pi R_M^2$ muons from the sample." The purpose of this note is to calculate the corrections to Eq. 1 to take into account the finite size of the moon.

Without the point-moon approximation, Eq. 1 clearly becomes

$$\frac{dN_\mu}{d\Omega} = \lambda\left(1 - \int_0^{R_M} r'dr' \int_0^{2\pi} d\theta \, \mathrm{G}(|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|, \sigma)\right), \tag{2}$$

where G is a two-dimensional Gaussian convolution. Writing G out explicitly,

$$\frac{dN_\mu}{d\Omega} = \lambda\left(1 - \frac{1}{2\pi\sigma^2} \int_0^{R_M} r'dr' \int_0^{2\pi} d\theta \, e^{-\frac{r^2 + r'^2 - 2rr'\cos\theta}{2\sigma^2}}\right). \tag{3}$$

Performing the θ integration, we obtain

$$\frac{dN_\mu}{d\Omega} = \lambda\left(1 - \frac{1}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \int_0^{R_M} r'dr' \, \mathrm{I}_0\!\left(\frac{rr'}{\sigma^2}\right) e^{-\frac{r'^2}{2\sigma^2}}\right), \tag{4}$$

where I_0 is a modified Bessel function of the first kind. The remaining integration in Eq. 4 cannot be done in closed form. However, I_0 has a simple and rapidly decreasing Taylor expansion,

$$\mathrm{I}_0(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{4^k (k!)^2} \; = \; 1 + \frac{x^2}{4} + \frac{x^4}{64} + \frac{x^6}{2304} \cdots. \tag{5}$$

Substituting the expansion and evaluating the integral, we obtain

$$\frac{dN_\mu}{d\Omega} = \lambda\left(1 - \frac{R_M^2}{2\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \left(1 + \frac{(r^2 - 2\sigma^2)R_M^2}{8\sigma^4} + \frac{(r^4 - 8r^2\sigma^2 + 8\sigma^4)R_M^4}{192\sigma^8} + \ldots\right)\right). \tag{6}$$

Using $R_M = 0.26^\circ$ and $\sigma = 0.41^\circ$, in our region of interest the first and second correction terms provide r-dependent corrections of up to -10% and +0.7%, respectively. The next term in the expansion (not shown) contributes up to -0.03%.

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[1] E. H. Cobb et al., *Phys. Rev. D* **61**, 092002 (2000).