Maximum Detectable Momentum in the MINOS Far Detector

T. Fields Feb 7, 2006

Abstract

Using a simple model, the maximum detectable momentum (MDM) in the FD is estimated as a function of the track impact parameter, polar angle ϑ_z, and length. In addition to the basic measuring accuracy, additional momentum accuracy limitations arising from multiple Coulomb scattering and from alignment errors are considered briefly.

Definition of MDM

Several definitions of MDM have been used in past decades. The one I shall use in this paper is: MDM is that momentum for which a typical nearly straight real track will have a measured curvature (determined from a fit to points along the track) which is one standard deviation from zero. The MDM is simply the reciprocal of the error (s.d.) of the curvature measurement (when the curvature is expressed in $(\mathrm{GeV/c})^{-1}$).

This definition means that, if the measured curvatures were to have a Gaussian distribution, a track whose measured momentum is equal to the MDM has an 86% probability of correct sign of charge, and an 86% probability of having a momentum greater that MDM/2.

Measuring the MDM

The usual recipe for measuring the MDM experimentally is this: Turn off the magnetic field, send a test beam of monoenergetic high momentum particles into the detector, and measure the curvature of the tracks. The resulting curvature distribution will have 3 kinds of sources: spatial measuring precision, errors (distortions) of the detector coordinate system, and multiple Coulomb scattering (MCS) of the particles in the detector. The first two sources are dependent upon the geometry and alignment. The MCS curvature will scale as 1/p and can be calculated accurately and absolutely. A nonzero average curvature will be evidence for curvature distortions of the detector and/or its coordinate system.

In our case, running with zero field poses some difficulties, and we do not have a monoenergetic high momentum test beam at the FD. We can nevertheless learn a lot about the FD MDM by studying high momentum cosmic muon tracks with the field off and also by running with reverse field. I have described how systematic errors in momentum determination can affect cosmic muon charge ratio measurements elsewhere [1]. In this note, I shall focus on making simple *a priori* estimates of MDM in the FD.

MDM Is Irrelevant for Stopping Tracks

Momentum measurements are very inaccurate for momenta which are comparable to the MDM. Furthermore, the B field in the iron plates is hard to measure accurately. Hence, for precise calibration of the FD curvature (momentum) scale, the use of stopping muon tracks is practically the only way to go. Stoppers are (almost) a completely separate subject and are not treated in this note.

Simple FD Model

One can study MDM effects in the FD using the full spatial reconstruction of the standard MINOS software chain, for both real and MC tracks. Many such studies have been done and are continuing.

In this note, I shall instead use a simplified detector model in an attempt to achieve additional useful insights into the basics of measuring high momentum tracks in the FD. This model can be refined for specific situations if needed.

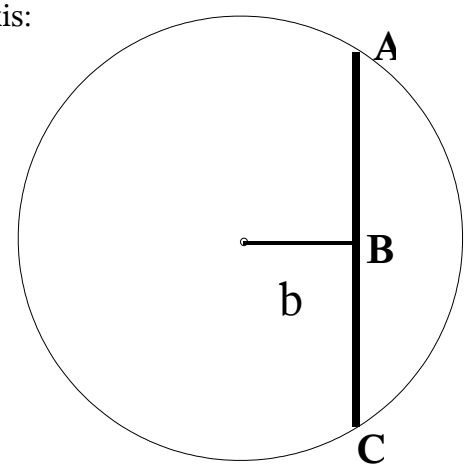
The simplifying assumptions that I have used are :

- *Geometry*: The detector is a right circular cylinder of radius R=4m, length L=29 m, and density=3.42 gm/cc.
- *Coordinate system:* +z along northward cylinder axis; +y up, +x westward.
- *B field:* uniform, azimuthal, B= 0.6 Tesla.
- *MCS:* Radiation length= 3.83 cm
- *Energy Loss*=0

Track Geometry and Impact Parameter

Muons whose momenta are comparable to the MDM are very straight. We shall visualize them as perfectly straight.

The following figure shows the situation projected onto a plane which is perpendicular to the z axis:



The track enters the detector at the point A and leaves at C. For clarity, we have made an azimuthal rotation of the coordinate system so that the track projection onto this plane is vertical. This rotation does not affect the generality of our results. (Aside – this generality depends on the fact that sagitta measurements in the u plane or v plane or any plane intermediate between them have the same accuracy. See Appendix 1.)

For now, I shall assume that the track enters and leaves through the cylindrical sides of the detector (a "side-to-side" track). Later I shall remark about tracks that begin or leave through the ends or that begin within the detector.

We define the impact parameter b as the distance of closest radial (cylindrical coordinate) approach of the track to the z axis. In fact, we shall now assume that the plane shown above contains the impact parameter line segment b. Then B is in the plane of the drawing, but A and C are not. The track AC penetrates this plane at the point B, which lies in the central (horizontal) plane of this rotated coordinate system. Thus the distances AB and BC are equal, i.e. B bisects the track. In what follows we shall concentrate on the

half-track AB, because the direction of the magnetic field is quite different for the two half-tracks AB and BC.

The remaining track coordinate of importance is the angle ϑ_z which ABC makes with the direction of the z axis. The actual length of the half-track AB is given by

(1) $$L(AB)=\sqrt{(R^2-b^2)}/\sin(\vartheta_z)$$ (half-length of a side-to-side track)

To calculate the precision of momentum measurement, we need the component of the magnetic field **B** which is perpendicular to the track direction. We approximate this by finding **B** at the midpoint of the half-track AB and then resolving it into components parallel and perpendicular to the track. We get

(2) $$B_{PERP}=B*\sqrt{[1-\sin^2(\vartheta_z)*\cos^2(\arctan(\sqrt{(R^2-b^2)}/(2*b)))]}$$

We now have expressions for the two basic quantities which determine the momentum measuring precision, namely the track length and the bending magnetic field. In the next section we shall give some numerical results.

Numerical Results for "side to side" Tracks

Zero Impact Parameter Cases:

In [2], I described the results of a simple MC calculation of least squares curvature fitting to hits in MINOS scintillator strips caused by a straight track. I have recently updated that calculation, mainly by using a corrected scintillator plane spacing, and keeping more careful account of number of hits, track length, and other quantities which depend on ϑ_z.

I now find the rms curvature error for an ensemble of 3.65 meter long, zero impact parameter straight tracks at ϑ_z=80 deg to be 1.2E-3 meter^{-1}. For smaller ϑ_z , and a track length of 3.93 m , this curvature error increases substantially to about 2.3E-3 meter^{-1} **,** because the high precision of the z coordinate is not so important when measuring the sagitta of horizontal tracks.

The corresponding radii of curvature are respectively 833 and 433 meters, corresponding to momenta of 149 and 77 GeV/c in our average Bfield of 0.60 Tesla.

*[As an aside, note that a curvature radius of 433 m corresponds to a sagitta of L*L/(8*rho) , which is 4.5 mm in this case. In other words, the sagitta of any reasonably horizontal and straight 4 meter track will be uncertain by a standard deviation of about 4.5 mm (due just to the finite height of each scintillator strip). A hand-waving a priori estimate of this sagitta might be something like:*

(3) $$(K)*[1/\sqrt{(12)}]*strip\ height/\sqrt{(number\ of\ planes\ hit)}$$,

where K is a factor >1 which expresses the fact that we are taking differences of average transverse coordinates. Taking a guess that K=2, this expression gives
2*0.29*41 mm/sqrt(36 hits) = 4 mm. *Not too bad.*]

In expression (3), we can see that increasing the track length will (increase the number of hits and thus) decrease the rms fitted sagitta error as $1/\sqrt{(L)}$. In addition, for a given (false) sagitta , the equivalent (false) radius of curvature is proportional to L^2 . The combined effect would be to increase (scale) the MDM proportional to $L^{2.5}$. (In fact, the track fit simulations yield a dependence of $L^{2.4}$, which I shall use in what follows.) So, scaling the MDM of the ϑ_z=80 deg half track from L=3.65 m to the required L=4.05 m increases its MDM from 149 to 191 GeV/c.

Finally, it is important to remember that the formulas given so far apply to one half of the track. The other half of the track will bend in the opposite direction and we shall measure its curvature independently. This will decrease the overall curvature error by $\sqrt{(2)}$ and thus increase the MDM by $\sqrt{(2)}$.

The above numbers and formulas allow us to calculate the MDM for b=0 side-to-aside tracks. The resulting values of MDM are given in the b=0 column of Table 1 below.

Nonzero b Cases: In this section we shall scale the MDM from zero impact parameter cases to nonzero impact parameter tracks, with results which are given in the other columns of Table 1. We are still considering only side-to side tracks. For b nonzero, the field direction is not perpendicular to the track, and the MDM will be reduced by a factor Bperp/B. We do the scaling by first using eqns (1) and (2) above to calculate L(AB) and Bperp. Then we scale the MDM (horizontally across the table from the b=0 values!) according to Bperp* $L^{2.4}$.

Table 1 MDM and Length for side-to-side Tracks in MINOS FD

MDM in GeV/c

ϑ_z (degrees)=	Impact Parameter b (meters) 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
10	471	470	469	468	467	466	465	464
20	471	469	465	460	455	451	447	246
30	471	467	458	447	391	298	197	92
40	344	333	304	262	213	159	103	47
50	235	226	203	171	135	98	62	28
60	232	223	197	162	124	87	52	22
70	231	221	193	156	115	77	44	17
80	270	257	223	177	128	82	43	15

Full Track Length in Meters

ϑ_z (degrees)=	Impact Parameter b (meters) 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
10	14.5	14.5	14.5	14.5	14.5	14.5	14.5	14.5
20	14.5	14.5	14.5	14.5	14.5	14.5	14.5	11.3
30	14.5	14.5	14.5	14.5	13.9	12.5	10.6	7.7
40	12.4	12.3	12.1	11.5	10.8	9.7	8.2	6.0
50	10.4	10.4	10.1	9.7	9.0	8.2	6.9	5.1
60	9.2	9.2	8.9	8.6	8.0	7.2	6.1	4.5
70	8.5	8.4	8.2	7.9	7.4	6.6	5.6	4.1
80	8.1	8.1	7.9	7.5	7.0	6.3	5.4	3.9

maxL=14.5 m=length of supermodule

Some things to note about Table 1:

- The variation in MDM is mainly due to the track length variation, and much less due to the change of Bperp. For reference, Table 2 below gives values of Bperp/B
- Because cosmic muons tracks are mainly vertical, they are usually what I have called side-to-side tracks in this paper. Hence the table applies directly to cosmic muons.
- The falloff of the Bfield with radius is not taken into account. This effect would decrease the MDM at large impact parameters to about 80% of the tabulated values.
- For ϑ_z= 90 deg, there are too few hits to measure the track curvature.
- Maximum track length has been defined as 14.5 m.

Table 2

Bperp/B

ϑ_z (degrees)=	Impact Parameter b (meters) 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
10	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99
20	1.00	1.00	0.99	0.98	0.97	0.96	0.95	0.94
30	1.00	0.99	0.97	0.95	0.93	0.91	0.89	0.88
40	1.00	0.99	0.96	0.91	0.87	0.84	0.81	0.79
50	1.00	0.98	0.94	0.88	0.82	0.76	0.71	0.67
60	1.00	0.98	0.92	0.84	0.76	0.68	0.61	0.55
70	1.00	0.97	0.90	0.81	0.70	0.60	0.51	0.42
80	1.00	0.97	0.89	0.79	0.67	0.55	0.43	0.31

Other Track Geometries

Other b=0 cases: All tracks which are parallel to the z axis (which would generally go from end-to-end or from within-to-end) are also b=0 tracks (that is, they also are everywhere perpendicular to the magnetic field). For such b=0 tracks, since the field along them does not reverse at the midpoint, it is clear from the above discussion and from the 30 deg b=0 entry in Table 1 that we should scale the MDM as $L^{2.4}$, starting from a (half-track) value of MDM=333 GeV/c for L=7.3 m. In this case there is no other half of the track with reversed field that we would measure independently.

The same arguments and scaling will apply to any track which travels mainly in the z direction, and is bent in a single direction by the full azimuthal Bfield.

An exception would be the case where an axial track traverses both supermodules, In this case, presumably the two segments of the track would in effect be measured independently. So the two values of MDM would be combined in quadrature. A similar situation would obtain for a paraxial track which crosses the coil hole (z axis).

Other b nonzero cases: We will not detail such cases here. The most crucial quantity is the track length. One can easily scale from the above cases to estimate the MDM for any nearly straight track whose length is greater than 2 meters or so.

Multiple Coulomb Scattering

In this well-studied process, many small-angle nuclear Coulomb scatterings combine to cause an angular deflection of an otherwise straight track. Projected onto a plane, the rms angle can be written as

(4) $$\theta_{MCS} = [0.0136/p(GeV/c)] * \sqrt{(L(meter)/X)} \quad ,$$

where X is the radiation length= 0.0383 meter in the FD.

The magnetic bending angle for this same length of track is given by

(5) $$\theta_{Bend} \approx L/Bending\ radius = L(meter) * B_{PERP}(Tesla)/[3.336 * p(GeV/c)] \quad .$$

Dividing, we have

(6) $$\theta_{MCS}/\theta_{Bend} = \frac{0.232}{[B_{PERP} * \sqrt{L}]} \qquad \text{(units as above).}$$

Note that p cancels from the ratio in this equation.

To avoid possible confusion about how to best deal with the opposite-bending second half, I have applied eqn 6 to only the first half-length of the side-to-side tracks that are listed in Table 1. Table 3 gives the value of the above ratio, as well as the track length, **for these half tracks**.

Table 3 - Multiple Coulomb Scattering

Theta$_{proj}$(MCS)/Theta(bend)

ϑ_Z (degrees)=	Impact Parameter b (meters) 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
10	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15
20	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.17
30	0.14	0.14	0.15	0.15	0.16	0.17	0.19	0.22
40	0.15	0.16	0.16	0.18	0.19	0.21	0.24	0.28
50	0.17	0.17	0.18	0.20	0.22	0.25	0.29	0.36
60	0.18	0.18	0.20	0.22	0.26	0.30	0.36	0.47
70	0.19	0.19	0.21	0.24	0.29	0.35	0.45	0.64
80	0.19	0.20	0.22	0.25	0.31	0.39	0.54	0.88

MCS Track Length in Meters

ϑ_Z (degrees)=	Impact Parameter b (meters) 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
10	7.3	7.3	7.3	7.3	7.3	7.3	7.3	7.3
20	7.3	7.3	7.3	7.3	7.3	7.3	7.3	5.7
30	7.3	7.3	7.3	7.3	6.9	6.2	5.3	3.9
40	6.2	6.2	6.0	5.8	5.4	4.9	4.1	3.0
50	5.2	5.2	5.1	4.8	4.5	4.1	3.5	2.5
60	4.6	4.6	4.5	4.3	4.0	3.6	3.1	2.2
70	4.3	4.2	4.1	3.9	3.7	3.3	2.8	2.1
80	4.1	4.0	3.9	3.8	3.5	3.2	2.7	2.0

We see that for most of the table entries, MCS is fairly small compared to magnetic bending. In those geometries, the expected accuracy of momentum measurement for high momenta will not be sensitive to MCS.

But we also see in Table 3 that MCS becomes nearly as large as magnetic bending at b=3.5 m, ϑ_z = 80 deg., L=2m, primarily due to the low value of Bperp. In that region, a good momentum measurement is not possible at any value of p.

In the general case, the momentum errors from measurement and from MCS should be combined in quadrature. Thus the MCS provides a floor on the momentum accuracy. {Reminder - For a side-to-side track, Table 3 is for one half of the track.)

Alignment Effects

It would be highly desirable to align the detector elements and coordinate system with sufficient precision throughout the detector to achieve the "intrinsic" values of MDM which are described above. It is doubtful that such good alignment precision has been achieved yet, but further study of cosmic muon data is needed to better understand the situation. Random errors in scintillator strip z locations of about 3 mm would roughly halve the tabulated MDM for steep cosmic muon tracks (see [2]).

In addition, there is evidence in the cosmic muon FD data for instrumental charge asymmetry effects which are probably due to systematic misalignments (curvatures) in the detector coordinate system [1,2,3,4]. Bearing in mind the 4 mm sagitta error obtained from expression (3) above , it is not hard to deduce [2] that a ***systematic*** sagitta error of 1 mm in a 4 m track would have clearly noticeable effects. Such systematic curvature errors can be canceled (statistically) in a charge ratio measurement by combining normal field data with reverse field data [1]. But such systematic errors will have undesirable effects upon the MDM, both conceptually and numerically. Perhaps the FD systematic curvatures can be mapped and then decreased by using more powerful alignment methods.

Acknowledgments

I am indebted to my FARDNOP colleagues at Argonne and Indiana [3,4] who have done the data analysis and developed many of the ideas used in this note. Of course, the opinions and calculations in this note are my own.

Appendix 1 – Tracks which are not in the vz Plane

The curvature fitting calculations which I described in [2] and which are used above on page 3 were done for tracks in the vz plane. If we confine this appendix to tracks with zero impact parameter, then in the general case, the track will lie in a plane which is between the vz and uz planes and which also contains the z axis. So the longitudinal coordinate will still be z. The transverse coordinate in the track plane, say w, will satisfy

$u=w*\cos(\eta)$ and $v=w*\sin(\eta)$, where η is the angle between the w and u planes.

The sagitta error will always be the same for track (or projected track) fits in the u and v planes, and its value will be just what was calculated in [2]. Of course, all that really

matters is the sagitta error in the bend (track) plane. In this case, calling the sagitta error in the w plane SEw , we can write

(7) $$(SEw)^2 = \cos^2(\eta) * (SEu)^2 + \sin^2(\eta) * (SEv^2) \quad ,$$

which then reduces to SEw=SEv.

References

[1] T. Fields, forthcoming version of MINOS note 1483
[2] T. Fields, Oct 05 talk, Minos note 1367; to be updated
[3] Brian Rebel's thesis (2004)
[4] Eric Beall's thesis (2005)