Systematic Errors in the Cosmic Muon Charge Ratio Measurement

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This note describes four types of systematic errors which can seriously affect the measurement of the underground muon charge ratio in the MINOS far detector. All four types have been observed and studied in the FD cosmic muon data (more accurately called "atmospheric muon" data). Issues which are involved in relating the underground charge ratio to the charge ratio at the surface are not addressed in this note.

Because the statistical error of our charge ratio measurements is less than 1%, we are attempting to detect, understand, and correct for systematic errors also at the better than 1% level. In general, we can do this only by making many careful studies of the data itself. As the work has progressed, we have also made considerable use of various simulations using Monte Carlo- generated tracks, including both full MC simulations and simple MC models.

A valuable byproduct of this precise work will be a better understanding of potential systematic errors in the FD for other MINOS experiments which depend upon momentum and charge sign measurements. In particular, to avoid loss of statistics in low rate MINOS measurements, it will be important to minimize the use of hard cuts when determining charge sign and momentum.

As one of our primary diagnostic tests, we have used the same criterion that Brian Rebel used in his thesis [1] - a plot of the measured charge ratio versus muon momentum is expected to be constant to perhaps 1% over the momentum range from about 5 GeV/c to about 100 GeV/c. The physical reasons for expecting this flatness are twofold:

1. The surface charge ratio is expected (based on surface measurements and model calculations) to vary little between say ~700 GeV and ~850 GeV , which are the approximate median surface energies for 5 GeV and 100 GeV vertical underground muons.
2. There is in either case a spread of more than 100 GeV in the energies lost by these individual muons in the rock.

(Later, when our calculations of muon propagation through the rock are more complete, we expect to be able to make a more precise quantitative statement about the expected flatness of that plot).

To date we have focused most of our analysis efforts on the four kinds of systematic errors which are listed in Table 1 on the next page:

Table 1. **Four Types of Systematic Errors in MINOS Charge Ratio**

Type	Description	Correct with B Reversal ?	Correct with MC ?
Acceptance	In each region of the detector, cuts may affect + and – tracks differently. Non-uniform illumination may prevent expected cancellations, e.g by detector north/south symmetry.	Yes	Yes, if illumination and detector geometry are accurate in MC
Curvature Resolution	A majority of atmospheric muon tracks are straight to within the detector resolution. Thus the measured charge ratio will be randomized toward unity at very high momenta.	No	Yes, but requires accurate knowledge of resolution
Curvature Bias	Small regional misalignments of the coordinate system can cause false curvature of straight tracks, and thus lead to biased values of +/- at high momenta.	Yes	No
Bad Track Points	Inclusion of a faulty outlier hit can give too low a momentum and a random sign of charge.	No	Mostly no

In what follows, I shall describe each of the above four types of systematic error and describe possible corrective actions for each.

Acceptance

A simple type of acceptance bias is caused by the fact that the "normal" magnet polarity has been chosen to focus (para-axial) north-bound negative muons toward the z axis of the detector. This increases the detector acceptance for such (negative) muons, and decreases the detector acceptance for the corresponding positive muons.

However, for nearly vertical downgoing northbound muons, this same field configuration has the opposite effect: Bending that negative muon toward the z axis will decrease the number of scintillator planes which it crosses, and thus it may fail say a 20 plane requirement which it would have survived if positive.

Of course, the sign of this selection bias will reverse for (the symmetric) southbound near- vertical downgoing muons. We might quickly conclude that there should be no net bias and that a MC calculation should confirm that conclusion. But note that if the northbound and southbound intensities are unequal (say from overburden asymmetries), then exact cancellation will not occur in the data. Thus the MC acceptance results will not apply to the charge ratio data unless the MC accurately matches the actual muon intensity distribution as well as the actual detector geometry.

Using a simple "acceptance only MC". we have calculated the size of these biases. As an example, Fig 1 shows typical results for 10 GeV muons which were generated with a cosmic ray distribution in zenith angle.

However, there is a more general, more accurate, and simpler method to correct for acceptance bias. That method is to also take data with the B field reversed. A first analysis of the reverse field data was presented by Bryan Rebel in September 2004.

The relevant (and very powerful) theorem is the following: If the B field is truly reversed everywhere, the actual particle trajectories of any given momentum within the detector will interchange between positive and negative muons. Note that errors in the reconstruction geometry and the B field map, as well as the acceptance effects described above, can then be precisely canceled out in the charge ratio (as long as the sign of the charge can be measured unambiguously). Note also that this error cancellation will occur locally (throughout the detector), and thus does not require spatial uniformity or symmetry of the detector or its illumination.

The Appendix shows how to derive the following formula for combining charge ratio data from forward and reverse field to compute the corrected (true) charge ratio:

$$r = \sqrt{(r_N * r_R)} \quad ,$$

where r_N is the apparent charge ratio for normal field and r_R is for reverse field. Also

described there is an important consistency check for the validity (applicability) of this formula.

As mentioned above, this procedure will also cancel out effects of field map errors. Such errors are likely to be more important for low muon momenta, where mu+ and mu- with the same initial conditions will follow rather different trajectories.

This same correction method works, not only for acceptance effects and field map effects, but also for curvature distortions in the detector coordinate system, as will be discussed below.

Data analyses carried out by both the Indiana group and the Argonne group are relying heavily upon the use of reverse field data.

Curvature Resolution

This kind of systematic error occurs for high momentum tracks whose measured curvature, although correct "on the average", may undergo a (Gaussian-like) fluctuation of significant probability which yields the incorrect sign of charge (and curvature). In the limit of exactly straight tracks and the absence of curvature biases, the fitted charge sign will be random, and thus at high enough apparent momentum, the apparent charge ratio will be unity.

This charge randomization of straight (high momentum) tracks will not be substantially affected by reversing the B field. So this effect must be studied and corrected for using Monte Carlo generated tracks. The crucial basic quantity which sets the momentum scale for this randomization is the curvature resolution for various straight track geometries. Fig 2 shows this high momentum randomization effect for two different momentum resolutions, using a very simple MC model. Of course, the real detector has a range of resolutions, rather than a single value.

I have described elsewhere [2] a Monte Carlo calculation which gives estimates of the FD momentum resolution, taking account of not only scintillator strip size, but also of random errors in scintillator alignment. The momentum resolution will depend upon impact parameter and track length, as I have described in my note about Maximum Detectable Momentum [3].

Of course, the full reconstructed track Monte Carlo should give us valuable information about the required charge ratio correction required to compensate for our limited momentum resolution. But such a correction will be too large for comfort above ~100 GeV.

A different way to cope with curvature resolution effects is to reject all tracks whose curvature is more than a chosen number of standard deviations from zero. Another alternative cut is to remove tracks whose momentum cannot be measured well, e.g. because of insufficient track length. In any case, the accuracy of the charge ratio measurement will still be poor above say 100 GeV/c.

Curvature Bias

Imagine that the supposedly "aligned" detector coordinate system actually has a non-zero average curvature in some region. Then truly straight tracks (infinite momentum tracks) in that region would be reconstructed on the average with that same curvature (but with opposite sign) – a curvature bias. There would then be a resulting bias in the measured charge ratio.

If we reverse the B field, this charge bias reverses, just like the acceptance biases discussed above. It can be corrected for in the same way, using the same equation.

A very simple MC calculation [2] shows that a surprisingly large spurious charge ratio will arise from a curvature bias whose magnitude is only 10% of the resolution (s.d) in curvature. See Fig 3. To eliminate such a small curvature bias would require an (impossibly?) precise alignment, as described in [2].

For reference purposes, Fig 4 shows the exponentially falling muon momentum spectrum which I have used in computing Fig 2 and Fig 3 of this note. It is different from that used in Ref 2, but the charge ratio systematic effects are quite similar for the two spectra.

I speculate that the east-west charge ratio asymmetry effect seen in the data arises from regional curvature biases in the east and west sides of the FD. However, it is not easy to visualize the likely geometry of such curvature biases. Perhaps they can be mapped using our (small!) set of Bfield-off data.

Bad Track Points

Inclusion in the track fit of an unrelated hit which lies far from a smooth fit to all of the other hits on the track will tend to give a track momentum which is much too low and a charge sign which is random. The obvious symptom of this kind of problem would be a falloff toward unity of the charge ratio at low momentum. Reversing the field should have little effect. So this charge randomization at low momentum should be quite distinct from any of the above types of systematic errors.

MC generated tracks would not be likely to show such behavior, unless caused by effects which may be included to some extent in the full MC - such as noise hits or faulty hits outside the detector.

At present, we do not have a reliable scheme for eliminating or correcting for this effect. Clearly, more careful selection of track fit precision might be effective. A minimum track length cut could also be help. Such studies are now in progress.

Acknowledgments

This note contains several of my own results, but most of the above ideas were originated by my FARDNOP colleagues at Argonne and Indiana.

Appendix How to Use Field-Reversed Data

Consider muons of a definite momentum striking a certain region of the detector and having a certain directional spread. The acceptance of the detector may well be different for + and – muons because of deflection of the muons by the magnetic field of the detector.

The observables are:
N+ , N- , counts recorded in time t_N ; N means Normal direction of Bfield
R+ , R- , counts recorded in time t_R ; R means Reverse direction of Bfield

Let a1 and a2 be acceptances for + and for - with Normal field direction.

Then it follows from Maxwell's equations that the acceptances with Reverse field are a1 for – and a2 for +.

If we now assume a charge ratio r for the true +/- population ratio, we can write two independent equations for the charge ratio in which the acceptances have cancelled:

$$r1 = (N+ / t_N) / (R- / t_R)$$

$$r2 = (R+ / t_R) / (N- / t_N)$$

These two equations should give the same value for r1 and r2, unless the analysis has somehow assigned incorrect charge sign to some of the muons. So any difference between r1 and r2 is evidence for residual systematic error. (Note that if we aim for a precision of better than say 1% jn this comparison, we must know the effective exposure times to ~0.5 % or better !)

One can easily eliminate the running times between the two equations to get:

$$r = \sqrt{(r_N * r_R)} \qquad [\ r_N \text{ is } N+ / N- \text{ and } r_R \text{ is } R+ / R-\ ]$$

In words, one should take the geometric, not the arithmetic, mean of charge ratios measured with the two field polarities. This difference in the means can be important numerically if the ratios r_N and r_R are not nearly equal to each other.

In addition to geometrical acceptance effects, the above argument about charge ratios applies to systematic curvature biases and also to some other kinds of charge dependent biases such as errors in the field map.

References

1. B. Rebel, PhD thesis, Indiana, 2004.
2. T. Fields, talk at Oct 05 collaboration meeting; Minos Note 1367.
3. T. Fields, MINOS note, Feb 2006, Maximum Detectable Momentum in the FD

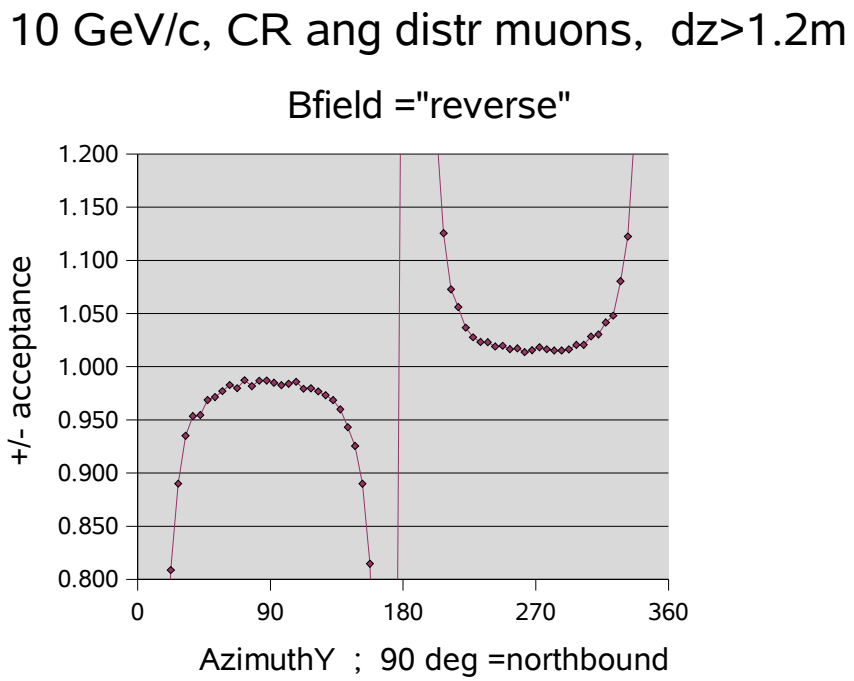

Fig 1. "Acceptance only" MC calculation for 10 GeV/c muons with a cosmic ray zenith angle distribution. Requiring dz>1.2m (i.e. hits in 20 or more planes) results in differences between positive and negative acceptance. Acceptance falls off toward zero for eastbound or westbound muons.

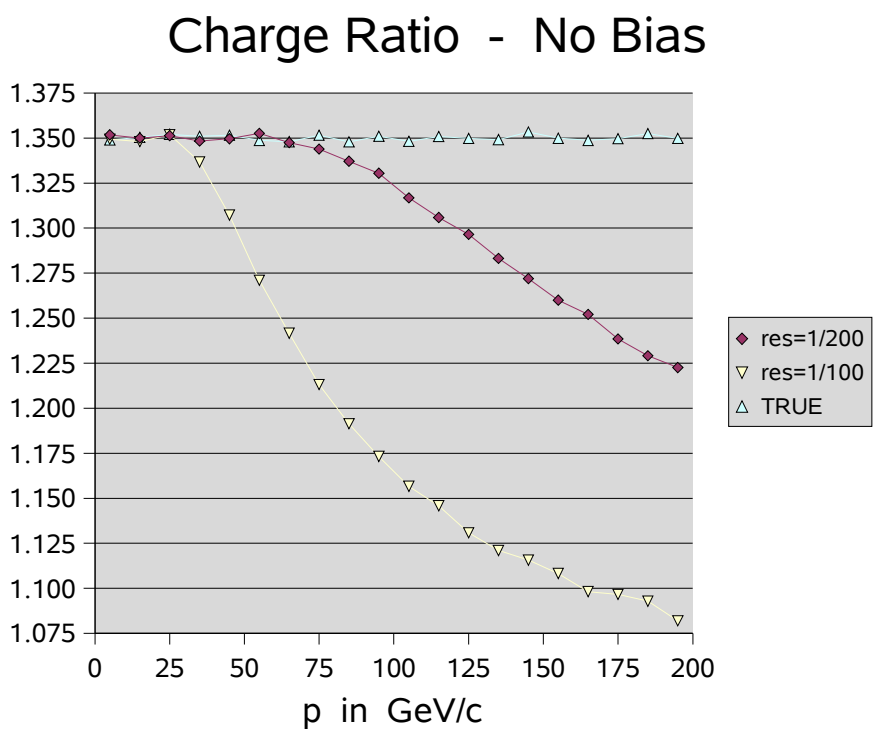

Fig. 2 Simple MC calculation of the apparent charge ratio as affected by the momentum resolution. The input charge ratio is 1.35 The symbols have the following meanings:

Middle curve: 1/200 means curvature resolution (sd) = 1/(200 GeV)
Lowest curve: 1/100 means curvature resolution (sd) = 1/(100 GeV)

According to Ref 3, roughly half of the far detector acceptance for cosmic muons should yield a one s.d. resolution of 1/200 GeV or somewhat better.

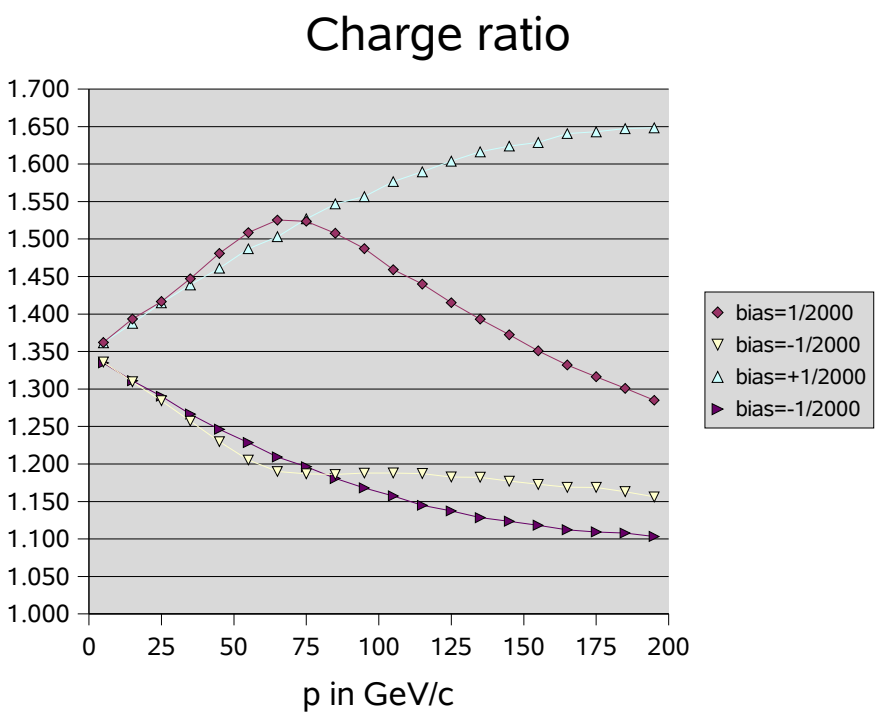

Fig 3 Charge ratio effects resulting from a constant curvature bias of 1/2000 GeV. The two curves which are highest and lowest at 200 GeV correspond to perfect (zero) resolution width. They apply to the two Bfield directions, and their geometric mean is 1.35 (the input value). The other two curves are for a curvature resolution of 1/200 GeV. Their geometric mean is equal to the middle curve in Fig 2.

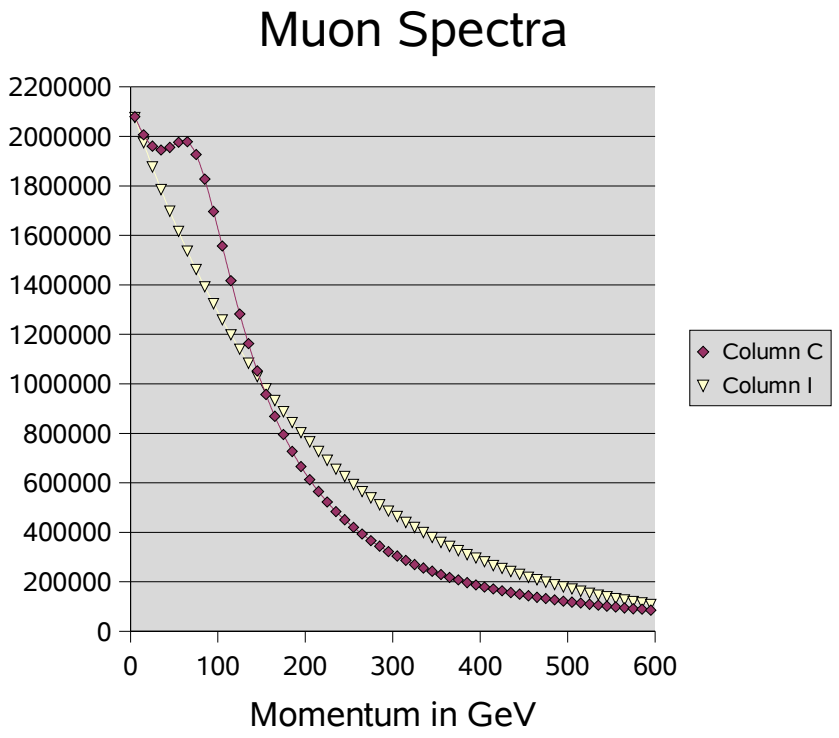

Fig 4 Assumed cosmic muon momentum spectrum in the Minos FD cavern. The curve which falls continuously is the assumed exponential true spectrum. Its mean momentum is 200 GeV. The curve with a secondary maximum near 65 GeV is the apparent (measured) spectrum if the curvature resolution is 1/200 GeV.