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Comments on Off-axis Beams

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ABSTRACT

This note discusses off-axis neutrino beams from a pedagogical point of view and then applies some of these ideas to the NuMI/MINOS situation.

1. Introduction.

The idea of neutrino off-axis beams was initially proposed for the BNL long baseline neutrino experiment. The basic motivation is to obtain a relatively intense and monoenergetic beam using neutrinos produced at a finite angle (ie off-axis) with respect to the direction of the hadronic beam. There has been recently a significant effort to apply some of these ideas to future neutrino programs, both in Japan and in USA, focusing especially on studies of $\nu_\mu \to \nu_e$ oscillations. I will start off this note by giving a simple pedagogical discussion of the relevant kinematics and why this idea has merit. This will be of interest primarily to those who have not thought much about this topic previously. Subsequently we shall look at how these ideas could be applied to long baseline neutrino experiments in general and to the NuMI beam line in particular and how fluxes and backgrounds would compare with a 0^o beam. We conclude by considering some practical considerations relevant to various options specific to the NuMI neutrino beam line.

Many of these points are discussed in considerable detail in NuMI-B-786 by Para and Schleper. That note presents a number of detailed Monte Carlo calculations relevant for off-axis beam mainly at a distance of 730 km. Our discussion here is somewhat more general, albeit more qualitative and pedagogical, attempting to understand the relevant issues for off-axis beams from elementary considerations.

2. Pion Decay Kinematics.

The principal source of ν's in an accelerator produced neutrino beam is the pion decay $\pi \to \mu + \nu$. The kinematics of the resulting neutrino in the laboratory system, which can be characterized by transverse and longitudinal momentum components, P_T and P_L, are determined by energy of the parent pion, characterized by γ, given by:

$$\gamma = E_\pi/m_\pi$$

the decay angle, θ*, in the pion rest frame with respect to its laboratory momentum vector and the neutrino momentum in the pion rest frame, p*, given by:

$$p^* = (m_\pi^2 - m_\mu^2)/(2\, m_\pi) = 29.8 \text{ MeV/c}.$$

The two laboratory momentum components are then given by:

$$P_T = p^* \sin\theta$$

$$P_L = \gamma\, p^* (1 + \cos\theta^*)$$

The locus of possible neutrino final states in the P_T, P_L space is given by an elliptical curve, where the laboratory momentum of the final state neutrino is proportional to the length of the vector from the origin to a specific P_T, P_L point and the laboratory angle is the angle of that vector with respect to the vertical axis. This is displayed in Fig 1 where we plot the decay kinematics for pion energies of 10, 15, and 20 GeV.

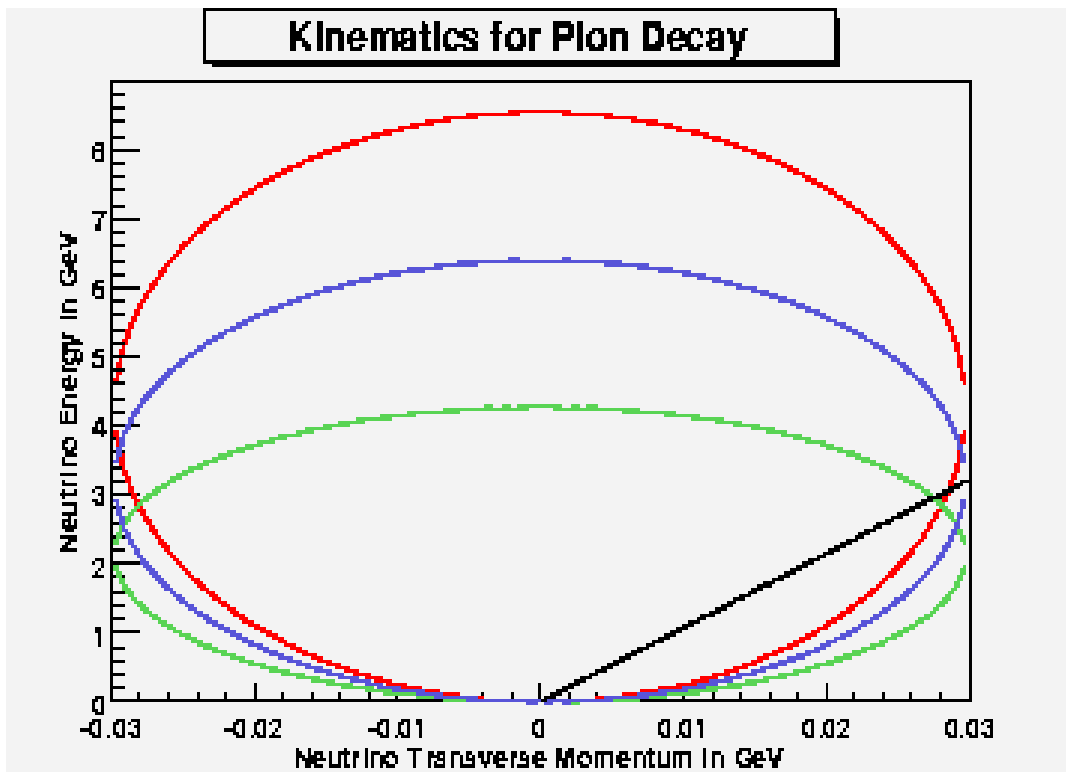

Fig.1. Pion decay kinematics for energies of 10 (green), 15 (blue), and 20 (red) GeV. The black line indicates the neutrino vector for a 90 deg decay for a pion of 15 GeV.

The optimum configuration for an off-axis beam is when we take a neutrino beam at a laboratory angle corresponding to $\theta^* = \pi/2$ and a pion energy such that we obtain thedesired neutrino energy from decays at that angle. These neutrinos have an energy ½ as large as those emitted in a forward direction at 0 degrees.This decay configuration is sometimes referred to as the "Jacobian peak" and its utility stems from the fact that that the final state laboratory configuration is relatively insensitive to variations in pion energy around the central value.

The validity of the above statement can be verified graphically in Fig 1. There we plot the neutrino decay vector at $\theta^* = \pi/2$ for 15 GeV pion. The energy (or momentum) of the neutrinos from 10 and 20 GeV pions, emitted at the same laboratory angle, will be given by the length of that 15 GeV vector from the origin to the intersection points with 10 and

20 GeV curves. As can be seen, the differences in length for the three cases are very small. We note from Fig.1 that the neutrino energy is somewhat lower for the other two energies, 10 and 20 GeV (due to sin θ^* being <1). Thus we would expect that the more precise optimization would push the optimum pion energy slightly higher than suggested by the arguments given here and hence the off-axis angle slightly lower.

We can also understand this mathematically from the expression for P_L, which for GeV neutrinos is essentially equivalent to the neutrino energy. Since sin θ^* changes very little around $\pi/2$, to maintain the same laboratory angle for higher (lower) pion energy, ie γ, we can decrease (increase) cos θ^* away from 0, so as to keep P_L (and hence neutrino energy) constant. Because of relative insensitivity of sin θ^* to changes around this value, P_T will not change significantly and thus if laboratory angle is forced to stay constant, that will automatically force the neutrino energy to stay constant.

3. Application to Long Baseline Experiments.

We consider here the relationship between L (distance from neutrino source to the detector), θ (off-axis laboratory angle) and E_π, the central pion energy for the energy band accepted by the focusing system. We derive a locus of optimal points in this space in zeroth order approximation. More detailed Monte Carlo calculations should not alter these conclusions significantly. Clearly, the θ - E_π relationship follows directly from our discussion above, ie (using small angle approximation):

$$\theta = P_T/P_L = 1/\gamma = m_\pi/E_\pi = 0.140/E_\pi \quad (E_\pi \text{ in GeV})$$

We shall assume that for performing $\nu_\mu \to \nu_e$ oscillation search, the optimum condition occurs when the oscillation is close to maximum since that maximizes the probability for oscillation into ν_e and minimizes potential sources of background. This occurs when oscillation phase is $\pi/2$, ie:

$$\pi/2 = 1.27\, L\, \Delta/E_\nu$$

where Δ is the neutrino mass squared difference, L is in km, and E_ν in GeV. This should be taken as zeroth order approximation. For $\theta^* = \pi/2$ decay we have from Section 2:

$$E_\nu = \gamma\, p^* = E_\pi\, p^*/m_\pi = 0.214\, E_\pi$$

And combining the last two equations we obtain:

$$1.57 = 1.27\, L\, \Delta / 0.214\, E_\pi \quad \text{or} \quad E_\pi = 3.78\, L\, \Delta$$

Using these expressions we can obtain a hyperbolic relation between L and θ, ie

$$0.140/\theta = 3.78\, L\, \Delta \quad \text{or} \quad L\, \theta = 0.140/3.78\, \Delta = 0.037/\Delta$$

Finally, we want to relate this last expression to the conditions imposed by a specific long baseline neutrino beam characterized by L_0 and λ, where L_0 is the distance from the source to the point where the beam comes back to the surface and λ is the initial dip angle (clearly these two are not independent). If we require the detector to be on or close to the surface, then L has to be bounded by

$$L_0 (1 - \theta/\lambda) < L < L_0 (1 + \theta/\lambda)$$

The extreme values of L occur when we take the neutrino beam from decays in a vertical plane. For a given desired value of θ, the locus of possible detector locations is an elliptical curve, the end points of the major axis corresponding to a beam from the decays in the vertical plane and the end points of the minor axis from transverse decays.

We proceed next to a discussion of specific features of these off-axis beams.

4. Fluxes.

We proceed next to compare fluxes in the region of interest for a 0^o and an off-axis beam. We consider in turn solid angle, bandwith of parent pions and relative numbers of parent pion decays,

Solid angle. The flux will be proportional to the solid angle in the pion rest frame, given by:

$$d\Omega = (A/4\pi L^2)\,(2\gamma/(1+\gamma^2\theta^2)^2$$

For our off-axis beam, $\gamma\theta = 1$ and γ is twice as large as for a forward beam for the same neutrino energy. Thus the solid angle factor cancels out.

Bandwidth of parent pions. As a basis for consideration we assume that we are interested in a looking at the full width energy spread of 20%, centered around our desired neutrino energy. At 0^o, the contributing parent energy spread will also be then 20%. For off-axis beam, the energies that will contribute are those that give $\sin\theta^* > 0.8$ for our chosen θ. This corresponds to $\cos\theta^* < 0.6$, and thus contributing pion energies span the region –50% to +100% around the mean energy. But since the pion energy is twice as high in this case as for 0^o, the ratio of ΔE_π's in GeV will be 2 x 150 / 20 = 15. Note that for such an energy spread the tuned pion energy should hence be about 10% higher than given by previous simple considerations to give us mean neutrino energy of interest..

Pion production probability. The NuMI neutrino interaction rate as a function of E_ν/GeV has roughly a linear dependence at low energies. But that factor includes enhancement by three powers of γ, two due to solid angle and one due to neutrino cross section. Thus pion production and decay probability goes roughly as $1/\gamma^2$.

Combining all the above factors would give us an overall gain in intensity in the desired energy band for an off-axis beam of a factor of 3.75. The actual calculations give more like a factor of 2 for flux enhancement. I believe that this is due to two causes: our focusing system cannot accept such a large energy spread (the full width of low and medium energy beams is about 80%) and parent pions have a finite angular spread which tends to smear out the neutrino energy at a particular laboratory angle.

5. Backgrounds.

We consider two principal backgrounds: NC events and ν_e's in the beam.

NC events. Inspection of Fig. 1 shows that at our chosen angle forward going pions *cannot* give a neutrino of a higher energy than the nominal ν energy regardless of pion energy. Thus the NC background in an off-axis beam from higher energy pions would vanish if they were all moving exactly at 0^o. Off-angle pions, however, can give higher energy neutrinos, especially when their angle with the beam axis is comparable to our chosen angle θ. Thus most of these high energy pion-produced neutrinos probably come from decays upstream.

The energy of neutrinos from K decays will be relatively insensitive to the decay angle since $\gamma_K \sim \gamma_\pi / 3.5$ and the neutrino energy is proportional to $1/(1+\gamma^2\theta^2)$. Thus this background should not change as we go to an off-axis beam.

Beam ν_e's. Since both γ and p* for μ's are very close to those of parent π's, the kinematics should neither suppress nor enhance this background in an off-axis beam. There may be some suppression due to the fact that with higher energy in the off-axis beam the probability of a decay in the beam pipe will be lower. I do not believe that this suppression is very large since most μ's will hit the beam pipe before decaying and not traverse the full length of the pipe.

The K^+ and K_L situation is more complex. With respect to 0^o beam neutrinos, ν_e's from these sources will have their solid angle suppressed by only 1.08 (as opposed to 4 for ν_μ from π decay) since $\gamma\theta$ is considerably smaller. The spread in their energy will be about twice as large, however, due to doubled energy of K's in an off-axis beam. Thus smaller fraction will fall into our signal region. Furthermore p_e* will now have to be lower by roughly a factor of 2 to give the same energy as the pion originated ν_μ peak reducing the backround in the signal region due to decay matrix element. There is also the question of potential difference in π/K ratio at these two energies. I would expect the ratio of this background in the two beams to be about unity. Clearly, the ν_μ/ν_e ratio would improve.

6. NuMI Situation.

The NuMI beam is characterized by L_0 = 747 km (the beam comes out of the ground 12 km downstream of Soudan) and λ = 58 mr. Furthermore, the range of interest for Δ is (2 – 4) x 10^{-3} eV^2 . In Fig 2 we plot the geometrical constraints for NuMI in the L, θ space. In addition, we display the L – θ curves for three different values of Δ in the region of interest corresponding to 2, 3, and 4 x 10^{-3} eV^2. The energy scale displayed on the right refers to Δ = 3 x 10^{-3} eV^2. For the lower and higher values of Δ, the energy scale should be multiplied by 2/3 and 4/3 respectively.

7. Optimization Issues for NuMI.

As can be seen from Fig.2 there is a reasonably large spectrum of distances and angles that might be suitable. Since the latest version of NuMI beam allows for a continuously varying energy of secondary particles we can consider E_π as a parameter one can optimize. I suspect that for distances ~500km or below, the pion energies are too low to give appreciable fluxes. Thus the relevant areas to consider for detector location for upward off-axis beam would be south of Lake Superior, possibly along Route 13 in the vicinity of Port Wing. Further north, one can think of a detector in Lake Superior or on the North Shore in the vicinity of Twin Harbors. For purely transverse decay beams the detector could be located near Rt 1, east or west of Soudan. Finally, the furthest locations that appear feasible are in Canada, east of Fort Frances, with access either on Route 502 or Route 11.

The optimum location(and hence pion energy) will depend on many factors, both scientific and non-scientific. The scientific ones will have to be evaluated more quantitatively by Monte Carlo calculations. Here we limit ourselves to just briefly enumerating some of the more important considerations on the assumption that the dominant physics to be investigated is ν_μ -> ν_e oscillations.

a) Fluxes. Solid angle factor is independent of energy. Neutrino cross section and larger energy band contributing argue for higher energy; pion prduction energy dependence for lower. Detailed Monte Carlo calculations are needed to see what is preferred from the flux point of view.

a) Background from π^0. This probably will not vary significantly over the energy range under consideration. Unfortunately, no good data exist on forward energetic π^0 production in NC ν interactions in this energy range.

b) Background from ν_e's in the beam. This may be somewhat smaller at high energies since contribution from muon decays might be slightly suppressed there as the decay length for pions around 12 GeV is comparable to the length of our decay volume. The contribution from τ decays is relatively insignificant.

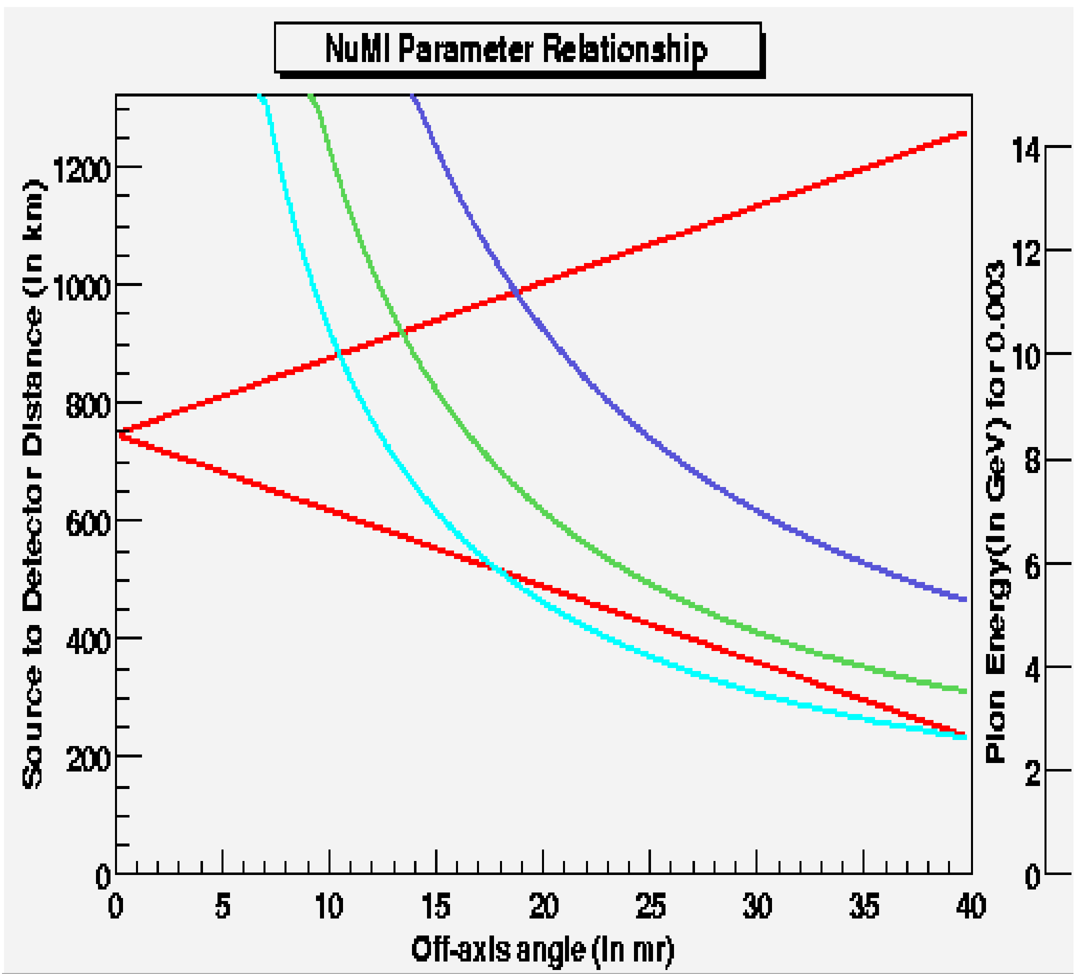

Fig. 2 – Relationship between optimum distance (L) and off-axis angle θ and 3 different values of Δ, namely 4, 3, and 2 x 10^{-3} eV^2 (turquoise, green and blue respectively). The allowed detector-on-the-surface-region for NuMI lies between the two red lines. The energy scale on the right corresponds to 3 x 10^{-3} eV^2.

c) Knowledge of background especially ν_e in the beam This is an important consideration as it may be the dominant systematic in the experiment. The contributions from the μ and K^+ decays can in principle be calculated from the neutrino rates and spectra in the Near Detector located on the beam axis. How well the produced K^+ flux can be estimated from the observed ν_μ radial and energy distributions is not clear to me at this time. The contributions from K_L's will be even more difficult to estimate. An off-axis Near Detector at an appropriate angle might be necessary to measure contributions from K decays. Since they come from almost a point source, the measurement should be quite accurate.

d) Detector issues. The design of the detector will most likely be coupled to the energy. For example, if one has to focus primarily on quasielastic events, a lower energy beam may be better.

e) Knowledge of Δm^2 . A better knowledge of that parameter will undoubtedly allow one to select between the different options in a more informed manner..

f) Geographical considerations. A more detailed study of the actual potential locations will help considerably in identifying the most appropriate sites.

g) Political considerations. There could be significant political and financial advantages in having a Canadian site.

8. Beam Optimization.

We consider how one might improve the NuMI beam to optimize it better for an off-axis beam. We consider three general areas.

Focusing. There is a *much higher* premium for optimized focusing in an off-axis beam then there is in an on-axis beam. Some of the specific gains are:

a) Wider energy band might be captured, all of which would contribute to the neutrinos of desired energy.
b) Pions would travel further and thus yields would be increased.
c) High energy tail from pion neutrinos would be decreased.
d) Since K^+'s decay quickly (in first ~100m for energies in question) and K_L's are unaffected by focusing, the ratio of π to K neutrinos would increase improving the signal to background ratio.
e) Since μ divergence is determined mainly by π kinematics and not by focusing, the of ν_e's from μ decay to the signal would also decrease.

Additional collimation. Some of our background (high energy neutrino tail from off-angle pions, ν_e's and high energy ν_μ's from K_L's) come from upstream region. Limiting the beam aperture downstream of the final horn could decrease these background sources.

Upstream bends. A potentially useful modification, though not practical for NuMI, might be to have 3 bending magnets just downstream of the horns, covering about 100 m, ie roughly a K decay length. The idea would be to create a zigzag path there which would significantly reduce neutrinos from K decay without significantly affecting the flux from pions. Whether such a concept has merit is not clear to me right now.

9. Conclusion. An off-axis NuMI beam in the 2-4 GeV range looks like a promising way to search for ν_μ -> ν_e oscillations. The desired ν_μ flux in the energy region of interest would be increased over that available in the 0^o beam. Both NC and ν_e backgrounds would be decreased (at least in the ratio to the signal). Additional modifications to the beam could improve the conditions even further.